

Egzamin z Mechaniki Konstrukcji, 6.2.2016
Exam on the Mechanics of Structures

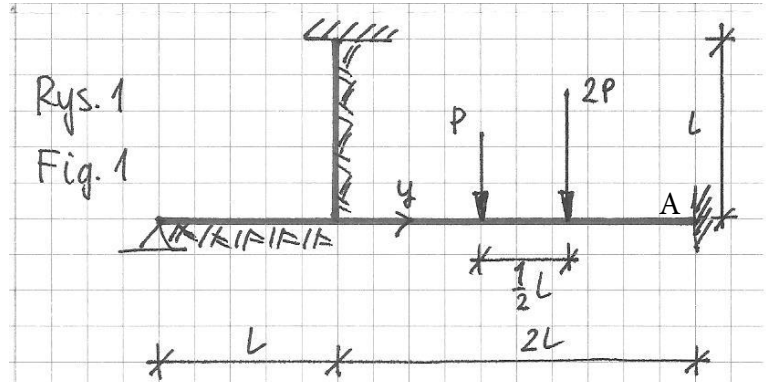
NAZWISKO, Imię LAST NAME, First Name				
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egzaminu	ocena łączna

Zadanie 1 (Rys. 1) Problem #1 (Fig. 1)

$EJ = \text{const. } \lambda = 1,4$

Wyznacz położenie sił $P, 2P$ tak, aby moment w podporze A był największy.

Find the position of forces $P, 2P$ for which the value of the moment at the support A is maximal.

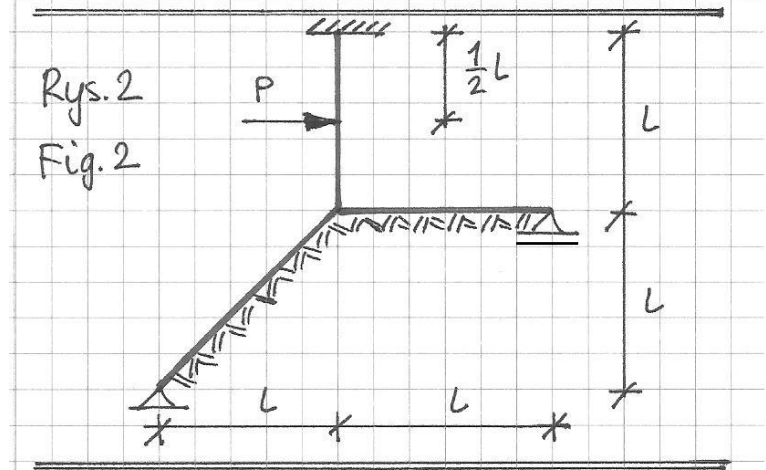


Zadanie 2 (Rys. 2) Problem #2 (Fig. 2)

$EJ = \text{const. } \lambda = 1,4$

Oblicz siłę podłużną w pręcie pochyłym.

Calculate the normal force in the inclined bar.

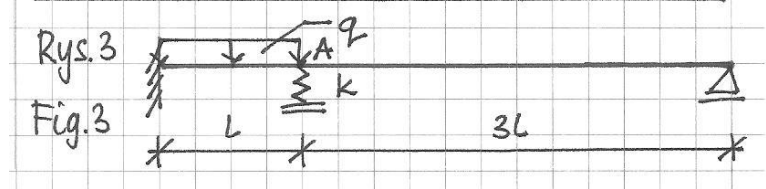


Zadanie 3 (Rys. 3) Problem #3 (Fig. 3)

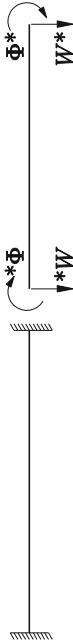
$EJ = \text{const.}$

Oblicz wartość współczynnika k , dla którego kąt obrotu przekroju nad podporą A wynosi zero.

Calculate the value of the coefficient k for which the rotation angle at the support A is zero.



λ	$\alpha(\lambda)$	$\beta(\lambda)$	$\vartheta(\lambda)$	$\delta(\lambda)$	$\gamma(\lambda)$	$\varepsilon(\lambda)$	$\alpha'(\lambda)$	$\vartheta'(\lambda)$	$\delta'(\lambda)$	$\gamma'(\lambda)$	$\varepsilon'(\lambda)$	$\chi(\lambda)$	$\alpha''(\lambda)$	$\vartheta''(\lambda)$	$\gamma''(\lambda)$	$\varepsilon''(\lambda)$	$\gamma'''(\lambda)$	$\varepsilon'''(\lambda)$
0,0	4,000	2,000	6,000	6,000	12,000	12,000	3,000	3,000	3,000	3,000	3,000	3,000	0,000	0,000	0,000	0,000	0,000	0,000
0,1	4,007	2,000	6,000	6,000	12,000	12,000	3,000	3,000	3,000	3,000	3,000	3,000	0,000	0,000	0,000	0,000	0,000	0,000
0,2	4,000	2,000	6,000	6,000	12,002	11,999	3,000	3,001	3,000	3,003	2,999	3,002	0,002	0,003	0,006	0,002	0,002	-0,001
0,3	4,000	2,000	6,002	5,999	12,012	11,996	3,001	3,003	3,001	3,016	2,995	3,008	0,011	0,016	0,032	0,011	0,011	-0,005
0,4	4,001	1,999	6,005	5,997	12,038	11,987	3,002	3,009	2,996	3,050	2,986	3,024	0,034	0,034	0,051	0,034	0,034	-0,017
0,5	4,002	1,998	6,013	5,992	12,093	11,968	3,005	3,021	2,990	3,121	2,965	3,059	0,082	0,123	0,247	0,083	0,083	-0,042
0,6	4,005	1,996	6,027	5,984	12,192	11,933	3,010	3,044	2,980	3,251	2,928	3,122	0,166	0,250	0,505	0,172	0,172	-0,086
0,7	4,009	1,993	6,050	5,970	12,356	11,877	3,018	3,082	2,962	3,465	2,867	3,226	0,298	0,449	0,918	0,318	0,318	-0,158
0,8	4,016	1,988	6,086	5,949	12,608	11,790	3,031	3,140	2,936	3,793	2,774	3,385	0,484	0,734	1,520	0,541	0,541	-0,268
0,9	4,025	1,981	6,137	5,919	12,972	11,665	3,050	3,223	2,898	4,267	2,639	3,615	0,726	1,107	2,340	0,861	0,861	-0,424
1,0	4,038	1,972	6,208	5,877	13,480	11,491	3,075	3,338	2,846	4,925	2,454	3,934	1,017	1,563	3,394	1,301	1,301	-0,635
1,1	4,055	1,959	6,304	5,821	14,163	11,258	3,109	3,492	2,776	5,807	2,209	4,363	1,342	2,087	4,688	1,884	1,884	-0,910
1,2	4,078	1,942	6,429	5,748	15,056	10,956	3,153	3,692	2,687	6,954	1,893	4,920	1,686	2,658	6,225	2,630	2,630	-1,253
1,3	4,107	1,920	6,589	5,656	16,197	10,573	3,209	3,944	2,575	8,408	1,500	5,626	2,031	3,258	8,008	3,560	3,560	-1,665
1,4	4,143	1,894	6,787	5,541	17,624	10,100	3,277	4,254	2,438	10,213	1,021	6,503	2,363	3,871	10,052	4,689	4,689	-2,144
1,5	4,186	1,862	7,030	5,402	19,377	9,526	3,359	4,629	2,275	12,408	0,455	7,571	2,675	4,492	12,380	6,029	6,029	-2,681
1,6	4,239	1,823	7,323	5,236	21,498	8,844	3,455	5,071	2,086	15,031	-0,200	8,847	2,963	5,119	15,027	7,587	7,587	-3,263
1,7	4,301	1,778	7,670	5,041	24,026	8,049	3,566	5,586	1,871	18,117	-0,941	10,350	3,228	5,756	18,031	9,368	9,368	-3,872
1,8	4,373	1,726	8,075	4,818	27,000	7,136	3,693	6,174	1,632	21,694	-1,759	12,093	3,472	6,411	21,438	11,372	11,372	-4,487
1,9	4,456	1,666	8,541	4,564	30,459	6,106	3,833	6,835	1,370	25,786	-2,642	14,088	3,700	7,092	25,290	13,599	13,599	-5,085
2,0	4,550	1,600	9,073	4,280	34,438	4,962	3,988	7,568	1,091	30,412	-3,573	16,347	3,915	7,807	29,631	16,049	16,049	-5,642



Schemat I

$$*\Phi = \frac{EJ}{l} \left(\alpha(\lambda) * \varphi + \beta(\lambda) \varphi^* + \vartheta(\lambda) \frac{w^*}{l} - \delta(\lambda) \frac{w^*}{l} \right)$$

$$\Phi^* = \frac{EJ}{l} \left(\beta(\lambda) * \varphi + \alpha(\lambda) \varphi^* + \delta(\lambda) \frac{w^*}{l} - \vartheta(\lambda) \frac{w^*}{l} \right)$$

$$*W = \frac{EJ}{l^2} \left(\vartheta(\lambda) * \varphi + \delta(\lambda) \varphi^* + \gamma(\lambda) \frac{w^*}{l} - \varepsilon(\lambda) \frac{w^*}{l} \right)$$

$$W^* = -\frac{EJ}{l^2} \left(\delta(\lambda) * \varphi + \vartheta(\lambda) \varphi^* + \varepsilon(\lambda) \frac{w^*}{l} - \gamma(\lambda) \frac{w^*}{l} \right)$$



Schemat II a

$$*\Phi = \frac{EJ}{l} \left(\alpha'(\lambda) * \varphi + \vartheta'(\lambda) \frac{w^*}{l} - \delta'(\lambda) \frac{w^*}{l} \right)$$

$$*\Phi = \frac{EJ}{l^2} \left(\vartheta'(\lambda) * \varphi + \gamma'(\lambda) \frac{w^*}{l} - \varepsilon'(\lambda) \frac{w^*}{l} \right)$$

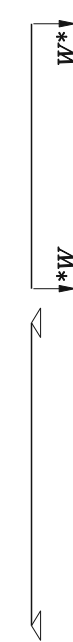
$$W^* = -\frac{EJ}{l^2} \left(\delta'(\lambda) * \varphi + \varepsilon'(\lambda) \frac{w^*}{l} - \chi'(\lambda) \frac{w^*}{l} \right)$$



Schemat III a

$$*\Phi = \frac{EJ}{l} \left(\alpha''(\lambda) * \varphi + \vartheta''(\lambda) \frac{w^*}{l} \right)$$

$$*W = \frac{EJ}{l^2} \left(\vartheta''(\lambda) * \varphi + \gamma''(\lambda) \frac{w^*}{l} \right)$$



Schemat IV

$$*W = \frac{EJ}{l^2} \left(\gamma'''(\lambda) \frac{w^*}{l} - \varepsilon'''(\lambda) \frac{w^*}{l} \right)$$

$$W^* = -\frac{EJ}{l^2} \left(\varepsilon'''(\lambda) \frac{w^*}{l} - \gamma'''(\lambda) \frac{w^*}{l} \right)$$

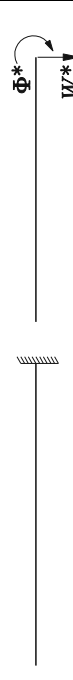


Schemat II b

$$\Phi^* = \frac{EJ}{l} \left(\alpha'(\lambda) \varphi^* + \delta'(\lambda) \frac{w^*}{l} - \vartheta'(\lambda) \frac{w^*}{l} \right)$$

$$*W = \frac{EJ}{l^2} \left(\delta'(\lambda) \varphi^* + \chi'(\lambda) \frac{w^*}{l} - \varepsilon'(\lambda) \frac{w^*}{l} \right)$$

$$W^* = -\frac{EJ}{l^2} \left(\vartheta'(\lambda) \varphi^* + \varepsilon'(\lambda) \frac{w^*}{l} - \gamma'(\lambda) \frac{w^*}{l} \right)$$



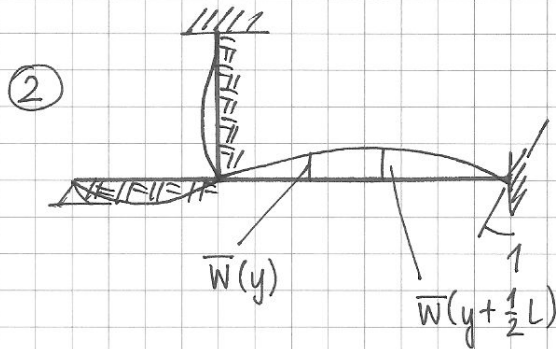
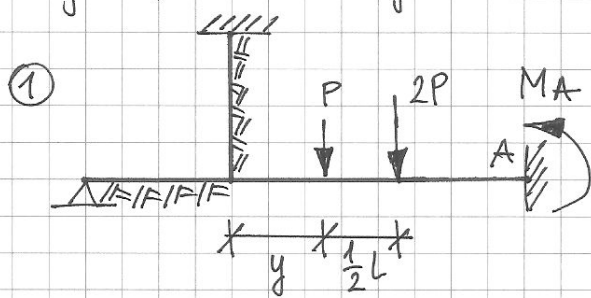
Schemat III b

$$\Phi^* = \frac{EJ}{l} \left(\alpha''(\lambda) \varphi^* - \vartheta''(\lambda) \frac{w^*}{l} \right)$$

$$W^* = -\frac{EJ}{l^2} \left(\vartheta''(\lambda) \varphi^* - \gamma''(\lambda) \frac{w^*}{l} \right)$$

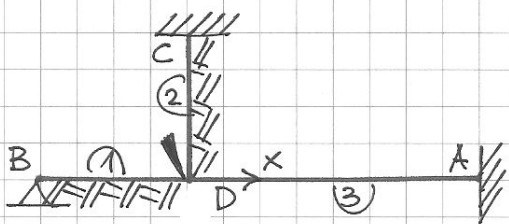
Egzamin z MK (IPB), 6 II 2016, zadanie 1

Ilustracja tw. Bettiego:



$$L_{12} = L_{21}$$

$$-M_A(y) \cdot 1 + P \cdot \bar{w}(y) + 2P \cdot \bar{w}(y + \frac{1}{2}L) = 0 \rightarrow M_A(y) = P \cdot \bar{w}(y) + 2P \cdot \bar{w}(y + \frac{1}{2}L)$$



Szukamy $\bar{w} = \bar{w}(x)$

stowarzyszonego z $\bar{\varphi}_A = 1$.

$$Q = [\varphi_D]$$

Równanie równowagi: $\Phi_D^{(1)} + \Phi_D^{(2)} + \Phi_D^{(3)} = 0$

Wzory transformacyjne

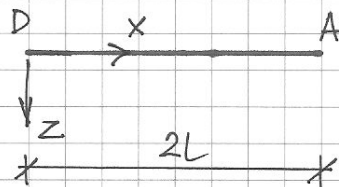
$$\Phi_D^{(1)} = \frac{EJ}{L} [\alpha'(1,4) \varphi_D]$$

$$\Phi_D^{(2)} = \frac{EJ}{L} [\alpha(1,4) \varphi_D]$$

$$\Phi_D^{(3)} = \frac{EJ}{2L} [4\varphi_D] + \frac{EJ}{2L} [2]$$

$$\varphi_D = -0,106$$

Linia ugięcia pręta D-A



$$\bar{w}(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3$$

$$\bar{w}(0) = 0$$

$$\bar{w}(2L) = 0$$

$$\frac{d\bar{w}}{dx} \Big|_{x=0} = \varphi_D$$

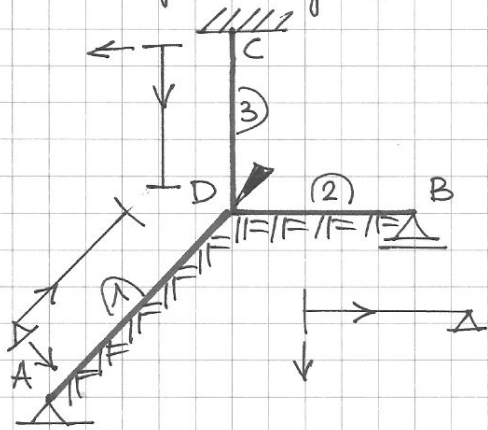
$$\frac{d\bar{w}}{dx} \Big|_{x=2L} = 1$$

$$\rightarrow \bar{w}(x) = L \cdot \left[1,788 \left(\frac{x}{2L} \right)^3 - 1,576 \left(\frac{x}{2L} \right)^2 - 0,212 \left(\frac{x}{2L} \right) \right]$$

$$\frac{dM_A}{dy} \Big|_{x=y} = 0 \rightarrow y = 0,923L \rightarrow |M_A^{\max}| = 0,867 PL$$

Egzamin z MK (IPB), 6 II 2016, zadanie 2

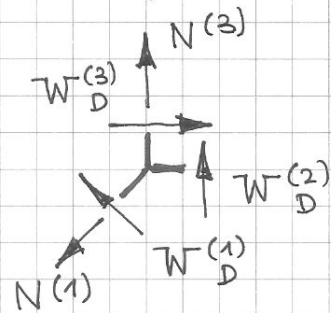
Schemat geometrycznie wyznaczalny



$$q_D = [\varphi_D]$$

Równowaga sił

w węzle D



$$N^{(1)} = -W_D^{(1)} + \sqrt{2} W_D^{(3)}$$

Równanie równowagi:

$$\Phi_D^{(1)} + \Phi_D^{(2)} + \Phi_D^{(3)} = 0$$

Wzory transformacyjne:

$$\Phi_D^{(1)} = \frac{EJ}{\sqrt{2}L} [\alpha'(1,4) \varphi_D]$$

$$\Phi_D^{(2)} = \frac{EJ}{L} [\alpha'(1,4) \varphi_D]$$

$$\Phi_D^{(3)} = \frac{EJ}{L} [4\varphi_D] - \frac{1}{8} PL$$

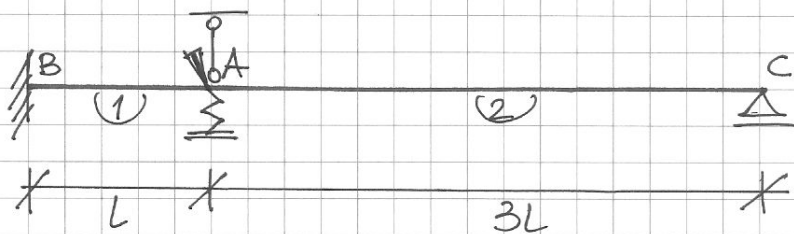
$$\varphi_D = 0,013 \frac{PL^2}{EJ}$$

$$W_D^{(1)} = -\frac{EJ}{2L^2} [\nu'(1,4) \varphi_D]$$

$$W_D^{(3)} = -\frac{EJ}{L^2} [6\varphi_D] - \frac{1}{2} P$$

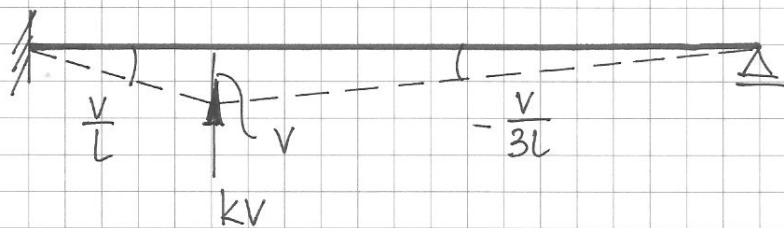
$$N^{(1)} = -0,789 P$$

Egzamin z MK (IPB), 6 II 2016, zadanie 3



$$q = \begin{bmatrix} \varphi_A \\ \frac{v}{L} \end{bmatrix}$$

Plan przesunięć:



Równania równowagi:

$$\Phi_A^{(1)} + \Phi_A^{(2)} = 0$$

$$[\Phi_A^{(1)} + \Phi_B^{(1)}] \cdot \frac{v}{L} + \Phi_A^{(2)} \cdot \left(-\frac{v}{3L}\right) - kvv + ql \cdot \frac{L}{2} \cdot \frac{v}{L} = 0$$

Wzory transformacyjne:

$$\Phi_A^{(1)} = \frac{EJ}{L} \left[4\varphi_A - 6\frac{v}{L} \right] + \frac{1}{12} ql^2$$

$$\Phi_B^{(1)} = \frac{EJ}{L} \left[2\varphi_A - 6\frac{v}{L} \right] - \frac{1}{12} ql^2$$

$$\Phi_A^{(2)} = \frac{EJ}{3L} \left[3\varphi_A + 3\frac{v}{3L} \right]$$

Stąd:

$$\begin{cases} 5\varphi_A - \frac{17}{3}\frac{v}{L} + \frac{1}{12}\frac{ql^3}{EJ} = 0 \\ -\frac{17}{3}\varphi_A + \left(\frac{109}{9} + \tau\right)\frac{v}{L} - \frac{1}{2}\frac{ql^3}{EJ} = 0 \end{cases}$$

$$\tau = \frac{kL^3}{EJ}$$

Otrzymujemy:

$$\varphi_A = \frac{(0,318 - 0,015\tau)}{5,015 + 0,882\tau} \frac{ql^3}{EJ}$$

$$\frac{v}{L} = 0,882\varphi_A + 0,015\frac{ql^3}{EJ}$$

$$\varphi_A = 0 \rightarrow \tau = 21,2 \rightarrow k = 21,2 \frac{EJ}{L^3}$$