Egzamin z Mechaniki Konstrukcji, 4.09.2015
Exam on the Mechanics of Structures

<table>
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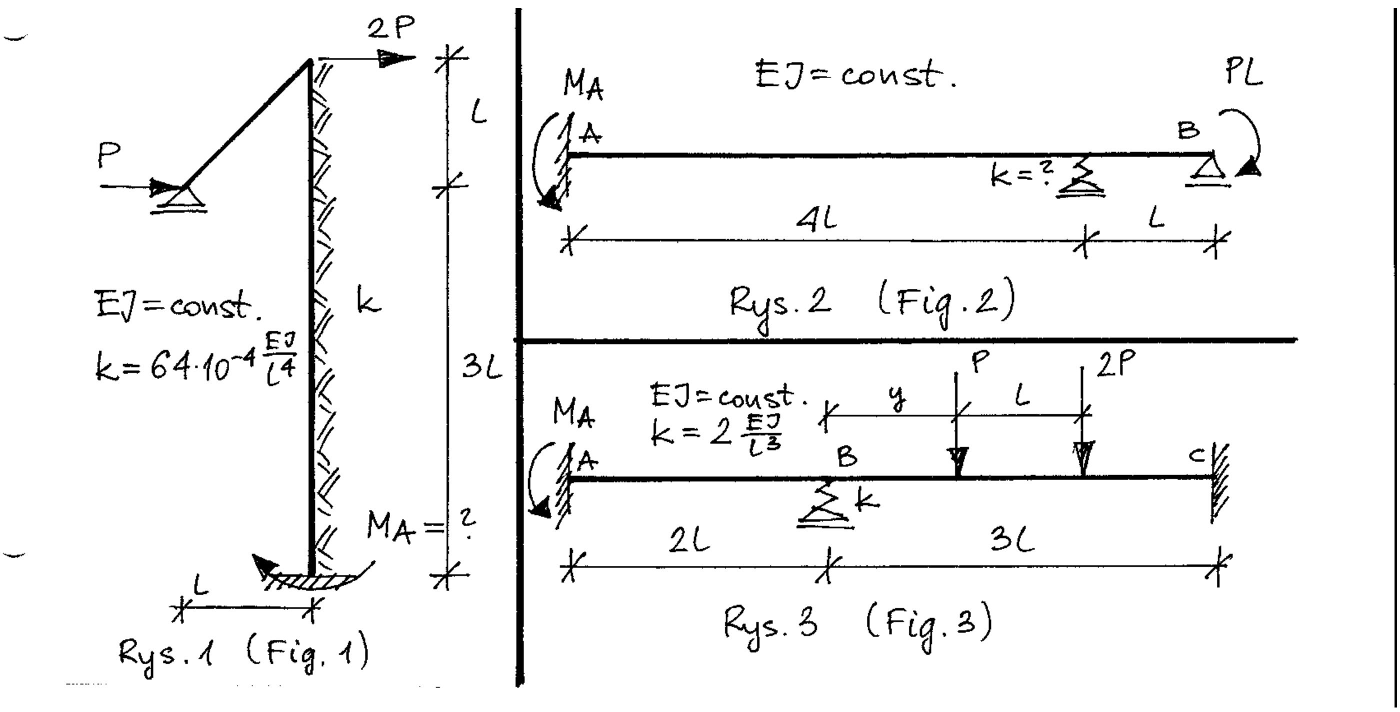
Zadanie 1 (Rys. 1) Problem #1 (Fig. 1)
Oblicz reakcję M_A korzystając z tw. Bettiego.
Calculate reaction M_A by the Betti Theorem.

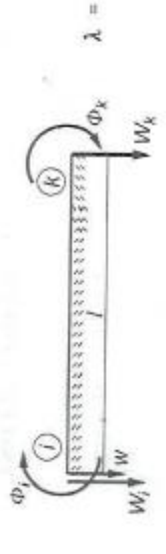
Zadanie 2 (Rys. 2) Problem #2 (Fig. 2)
Oblicz wartość k, dla której wartość $M_A = 0$. Następnie oblicz wartość kąta φ_B korzystając ze wzoru Maxwella-Mohra.
Calculate the value of k for which $M_A = 0$. Next, calculate φ_B by the Maxwell-Mohr formula.

Zadanie 3 (Rys. 3) Problem #3 (Fig. 3)
Wyznacz położenie układu sił (P, $2P$) w przedziale B-C tak, aby wartość bezwzględna $|M_A|$ była największa.
Find the position of force system (P, $2P$) between points B and C for which the absolute value $|M_A|$ is maximal.



λ	C(λ)	S(λ)	T(λ)	R(λ)	M(λ)	N(λ)	C'(λ)	T'(λ)	R'(λ)
0,0	**4,000**	**2,000**	**6,000**	**6,000**	**12,000**	**12,000**	**3,000**	**3,000**	**3,000**
0,1	4,000	2,000	6,000	6,000	12,000	12,000	3,000	3,000	3,000
0,2	4,000	2,000	6,000	6,000	12,002	11,999	3,000	3,001	3,000
0,3	4,000	2,000	6,002	5,999	12,012	11,996	3,001	3,003	2,999
0,4	4,001	1,999	6,005	5,997	12,038	11,987	3,002	3,009	2,996
0,5	**4,002**	**1,998**	**6,013**	**5,992**	**12,093**	**11,968**	**3,005**	**3,021**	**2,990**
0,6	4,005	1,996	6,027	5,984	12,192	11,933	3,010	3,044	2,980
0,7	4,009	1,993	6,050	5,970	12,356	11,877	3,018	3,082	2,962
0,8	4,016	1,988	6,086	5,949	12,608	11,790	3,031	3,140	2,936
0,9	4,025	1,981	6,137	5,919	12,972	11,665	3,050	3,223	2,898
1,0	**4,038**	**1,972**	**6,208**	**5,877**	**13,480**	**11,491**	**3,075**	**3,338**	**2,846**
1,1	4,055	1,959	6,304	5,821	14,163	11,258	3,109	3,492	2,776
1,2	4,078	1,942	6,429	5,748	15,056	10,956	3,153	3,692	2,687
1,3	4,107	1,920	6,589	5,656	16,197	10,573	3,209	3,944	2,575
1,4	4,143	1,894	6,787	5,541	17,624	10,100	3,277	4,254	2,438
1,5	**4,186**	**1,862**	**7,030**	**5,402**	**19,377**	**9,526**	**3,359**	**4,629**	**2,275**
1,6	4,239	1,823	7,323	5,236	21,498	8,844	3,455	5,071	2,086
1,7	4,301	1,778	7,670	5,041	24,026	8,049	3,566	5,586	1,871
1,8	4,373	1,726	8,075	4,818	27,000	7,136	3,693	6,174	1,632
1,9	4,456	1,666	8,541	4,564	30,459	6,106	3,833	6,835	1,370
2,0	**4,550**	**1,600**	**9,073**	**4,280**	**34,438**	**4,962**	**3,988**	**7,568**	**1,091**

λ	M'(λ)	N'(λ)	L'(λ)	C''(λ)	T''(λ)	M''(λ)	M'''(λ)	N'''(λ)
0,0	**3,000**	**3,000**	**3,000**	**0,000**	**0,000**	**0,000**	**0,000**	**0,000**
0,1	3,000	3,000	3,000	0,000	0,000	0,000	0,000	0,000
0,2	3,003	2,999	3,002	0,002	0,003	0,006	0,002	-0,001
0,3	3,016	2,995	3,008	0,011	0,016	0,032	0,011	-0,005
0,4	3,050	2,986	3,024	0,034	0,051	0,102	0,034	-0,017
0,5	**3,121**	**2,965**	**3,059**	**0,082**	**0,123**	**0,247**	**0,083**	**-0,042**
0,6	3,251	2,928	3,122	0,166	0,250	0,505	0,172	-0,086
0,7	3,465	2,867	3,226	0,298	0,449	0,918	0,318	-0,158
0,8	3,793	2,774	3,385	0,484	0,734	1,520	0,541	-0,268
0,9	4,267	2,639	3,615	0,726	1,107	2,340	0,861	-0,424
1,0	**4,925**	**2,454**	**3,934**	**1,017**	**1,563**	**3,394**	**1,301**	**-0,635**
1,1	5,807	2,209	4,363	1,342	2,087	4,688	1,884	-0,910
1,2	6,954	1,893	4,920	1,686	2,658	6,225	2,630	-1,253
1,3	8,408	1,500	5,626	2,031	3,258	8,008	3,560	-1,665
1,4	10,213	1,021	6,503	2,363	3,871	10,052	4,689	-2,144
1,5	**12,408**	**0,455**	**7,571**	**2,675**	**4,492**	**12,380**	**6,029**	**-2,681**
1,6	15,031	-0,200	8,847	2,963	5,119	15,027	7,587	-3,263
1,7	18,117	-0,941	10,350	3,228	5,756	18,031	9,368	-3,872
1,8	21,694	-1,759	12,093	3,472	6,411	21,438	11,372	-4,487
1,9	25,786	-2,642	14,088	3,700	7,092	25,290	13,599	-5,085
2,0	**30,412**	**-3,573**	**16,347**	**3,915**	**7,807**	**29,631**	**16,049**	**-5,642**



$$\lambda = l\sqrt[4]{\frac{k}{4EJ}}$$

$$\Phi_i = \frac{EJ}{l}\left[C(\lambda)\varphi_i + S(\lambda)\varphi_k + T(\lambda)\frac{w_i}{l} - R(\lambda)\frac{w_k}{l}\right] - \gamma(\lambda)pl^2$$

$$\Phi_k = \frac{EJ}{l}\left[S(\lambda)\varphi_i + C(\lambda)\varphi_k + R(\lambda)\frac{w_i}{l} - T(\lambda)\frac{w_k}{l}\right] + \gamma(\lambda)pl^2$$

$$W_i = \frac{EJ}{l^2}\left[T(\lambda)\varphi_i + R(\lambda)\varphi_k + M(\lambda)\frac{w_i}{l} - N(\lambda)\frac{w_k}{l}\right] - \gamma_T(\lambda)pl$$

$$W_k = -\frac{EJ}{l^2}\left[R(\lambda)\varphi_i + T(\lambda)\varphi_k + N(\lambda)\frac{w_i}{l} - M(\lambda)\frac{w_k}{l}\right] - \gamma_T(\lambda)pl$$

$$\Phi_i = \frac{EJ}{l}\left[C''(\lambda)\varphi_i + T''(\lambda)\frac{w_i}{l}\right]$$

$$W_i = \frac{EJ}{l^2}\left[T''(\lambda)\varphi_i + M''(\lambda)\frac{w_i}{l}\right]$$

$$\Phi_i = \frac{EJ}{l}\left[C'(\lambda)\varphi_i + T'(\lambda)\frac{w_i}{l} - R'(\lambda)\frac{w_k}{l}\right]$$

$$W_i = \frac{EJ}{l^2}\left[T'(\lambda)\varphi_i + M'(\lambda)\frac{w_i}{l} - N'(\lambda)\frac{w_k}{l}\right]$$

$$W_k = -\frac{EJ}{l^2}\left[R'(\lambda)\varphi_i + N'(\lambda)\frac{w_i}{l} - L'(\lambda)\frac{w_k}{l}\right]$$

$$\Phi_i = \frac{EJ}{l}\left[C''(\lambda)\varphi_k - T''(\lambda)\frac{w_k}{l}\right]$$

$$W_k = -\frac{EJ}{l^2}\left[T''(\lambda)\varphi_k - M''(\lambda)\frac{w_k}{l}\right]$$

$$\Phi_k = \frac{EJ}{l}\left[C'(\lambda)\varphi_k + R'(\lambda)\frac{w_i}{l} - T'(\lambda)\frac{w_k}{l}\right]$$

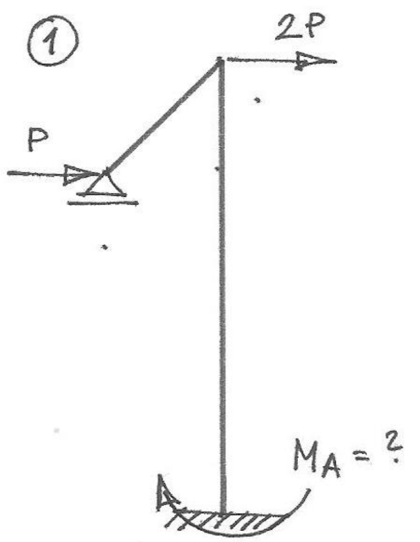
$$W_i = \frac{EJ}{l^2}\left[R'(\lambda)\varphi_k + L'(\lambda)\frac{w_i}{l} - N'(\lambda)\frac{w_k}{l}\right]$$

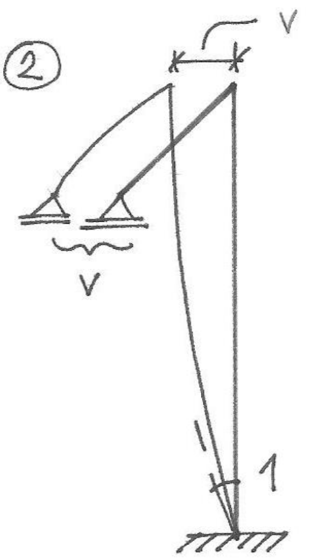
$$W_k = -\frac{EJ}{l^2}\left[T'(\lambda)\varphi_k + N'(\lambda)\frac{w_i}{l} - M'(\lambda)\frac{w_k}{l}\right]$$

$$W_i = \frac{EJ}{l^2}\left[M'''(\lambda)\frac{w_i}{l} - N'''(\lambda)\frac{w_k}{l}\right]$$

$$W_k = -\frac{EJ}{l^2}\left[N'''(\lambda)\frac{w_i}{l} - M'''(\lambda)\frac{w_k}{l}\right]$$

Egzamin z MK IPB, 4.09.2015, zadanie 1





Z tw. Bettiego:

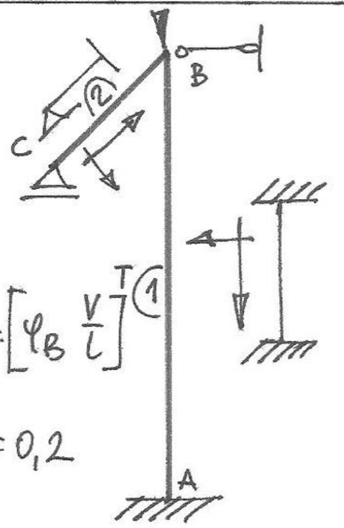
By the Betti Theorem:

$$-M_A \cdot 1 - 2P \cdot v - P \cdot v = 0$$

$$M_A = -3P \cdot v$$

gdzie v jest przesunięciem spowodowanym obrotem w p. A o kąt = 1 przeciwnie do zwrotu M_A.

where v stands for the translation related to the rotation angle = 1 at A opposite to the direction of M_A.



$$q = \left[\varphi_B \ \frac{V}{L}\right]^T$$

$$\lambda = 0{,}2$$

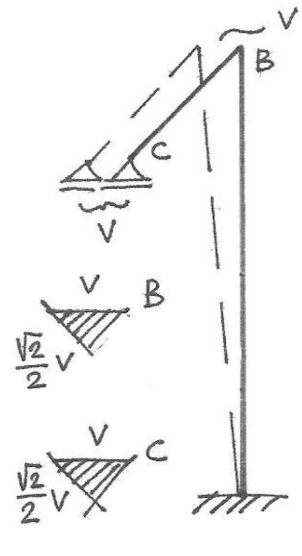
$$\Phi_B^{(1)} + \Phi_B^{(2)} = 0$$

$$W_B^{(1)} \cdot \bar{v} + W_B^{(2)} \cdot \left(-\frac{\sqrt{2}}{2}\bar{v}\right) + W_C^{(2)} \cdot \left(-\frac{\sqrt{2}}{2}\bar{v}\right) = 0$$

$$\Phi_B^{(1)} = \frac{EJ}{4L}\left[C(0{,}8)\varphi_B + T(0{,}8)\frac{V}{4L}\right] + \frac{EJ}{4L}\left[S(0{,}8)(-1)\right]$$

$$\Phi_B^{(2)} = \frac{3EJ}{\sqrt{2}L}\left[\varphi_B\right]$$

$$W_B^{(1)} = \frac{EJ}{(4L)^2}\left[T(0{,}8)\varphi_B + M(0{,}8)\frac{V}{4L}\right] + \frac{EJ}{(4L)^2}\left[R(0{,}8)(-1)\right]$$



$$W_B^{(2)} = -W_C^{(2)} = -\frac{3EJ}{2L^2}\left[\varphi_B\right]$$

$$\frac{EJ}{L}\begin{bmatrix} 3{,}125 & 0{,}38 \\ 0{,}38 & 0{,}197 \end{bmatrix}\begin{bmatrix} \varphi_B \\ \frac{V}{L} \end{bmatrix} - \frac{EJ}{L}\begin{bmatrix} 0{,}497 \\ 0{,}372 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\varphi_B = -0{,}092 \qquad \frac{V}{L} = 2{,}067$$

$$M_A = -6{,}197\, PL$$

opracował:
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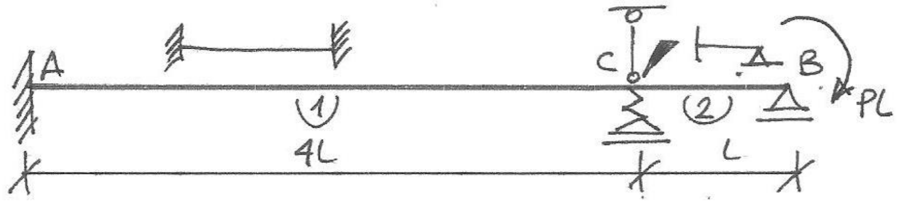
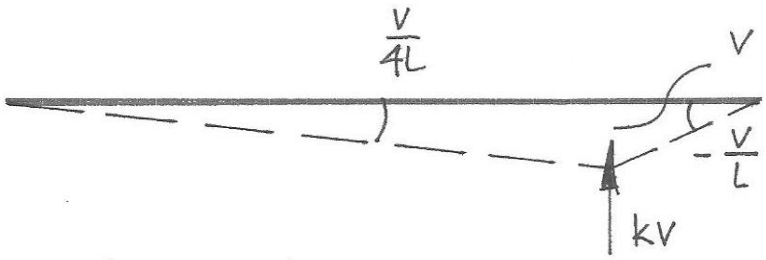
Egzamin z MK IPB, 4.09.2015, zadanie 2



$$q = \begin{bmatrix} \varphi_C \\ \frac{V}{L} \end{bmatrix}$$



$$\begin{cases} \Phi_C^{(1)} + \Phi_C^{(2)} = 0 \\ \left[\Phi_A^{(1)} + \Phi_C^{(1)}\right] \cdot \frac{\bar{V}}{4L} + \Phi_C^{(2)} \cdot \left(-\frac{\bar{V}}{L}\right) - kV\bar{V} + PL \cdot \left(-\frac{\bar{V}}{L}\right) = 0 \end{cases}$$

$$\begin{cases} \Phi_C^{(1)} + \Phi_C^{(2)} = 0 \\ -\frac{1}{4}\Phi_A^{(1)} - \frac{1}{4}\Phi_C^{(1)} + \Phi_C^{(2)} + kVL + PL = 0 \end{cases}$$

$$\Phi_A^{(1)} = \frac{2EJ}{4L}\left[\varphi_C - \frac{3}{4}\frac{V}{L}\right]$$

$$\Phi_C^{(1)} = \frac{2EJ}{4L}\left[2\varphi_C - \frac{3}{4}\frac{V}{L}\right]$$

$$\Phi_C^{(2)} = \frac{3EJ}{L}\left[\varphi_C + \frac{V}{L}\right] + \frac{1}{2}PL$$

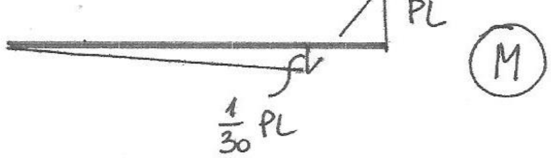
opracował: G. Dzierżanowski

$$\frac{EJ}{L}\begin{bmatrix} 4 & \frac{21}{8} \\ \frac{21}{8} & \frac{51}{16} + \tau \end{bmatrix}\begin{bmatrix} \varphi_B \\ \frac{V}{L} \end{bmatrix} - \begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \end{bmatrix} PL = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \tau = \frac{kL^3}{EJ}$$

$$\varphi_B = \frac{2(75 - 16\tau)}{375 + 256\tau}\frac{PL^2}{EJ} \quad ; \quad \frac{V}{L} = -\frac{300}{375 + 256\tau}\frac{PL^2}{EJ}$$

$$M_A = -\Phi_A^{(1)} = 0 \iff \tau = \frac{375}{32}$$

Na mocy tw. redukcyjnego:



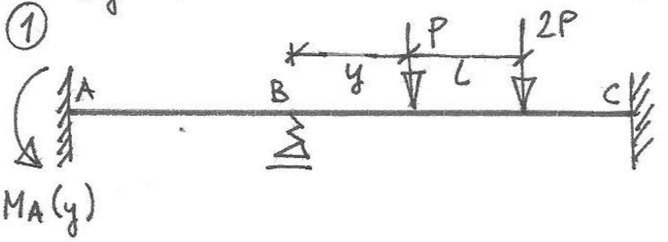
$$\varphi_B = \frac{59}{180}\frac{PL^2}{EJ}$$

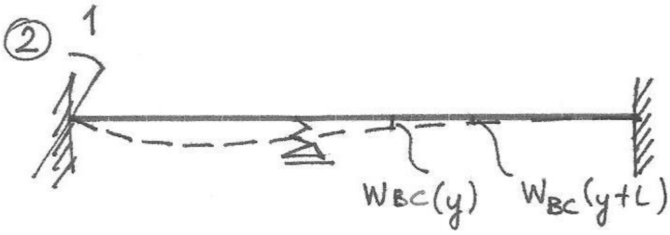
Egzamin z MK IPB, 4.09.2015, zadanie 3

Na mocy tw. Bettiego:

By the Betti Theorem:

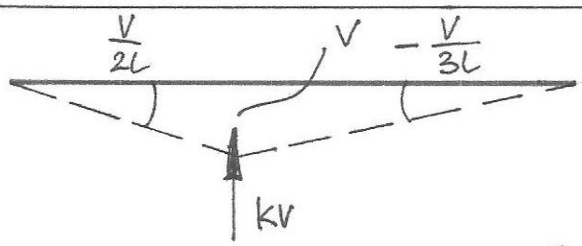
$$L_{12} = L_{21}$$

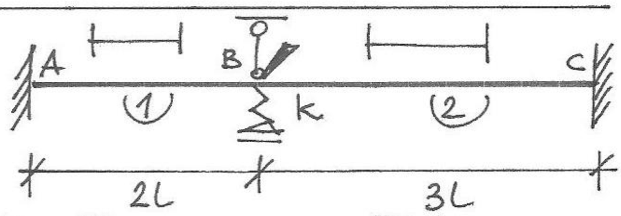




$$L_{12} = L_{21} \longrightarrow -M_A(y)\cdot 1 + P\cdot w_{BC}(y) + 2P\cdot w_{BC}(y+L) = 0$$

$$M_A(y) = P\cdot w_{BC}(y) + 2P\cdot w_{BC}(y+L)$$





$$q = \left[\varphi_B \;\; \frac{V}{L}\right]^T \qquad k = 2\frac{EJ}{L^3}$$

$$\begin{cases} \Phi_B^{(1)} + \Phi_B^{(2)} = 0 \\ \left[\Phi_A^{(1)} + \Phi_B^{(1)}\right]\cdot\frac{\bar{V}}{2L} + \left[\Phi_B^{(2)} + \Phi_C^{(2)}\right]\cdot\left(-\frac{\bar{V}}{3L}\right) - kV\bar{V} = 0 \end{cases}$$

$$\Phi_A^{(1)} = \frac{2EJ}{2L}\left[\varphi_B - 3\frac{V}{2L}\right] + \frac{2EJ}{2L}\left[2\right]$$

$$\Phi_B^{(1)} = \frac{2EJ}{2L}\left[2\varphi_B - 3\frac{V}{2L}\right] + \frac{2EJ}{2L}\left[1\right]$$

$$\Phi_B^{(2)} = \frac{2EJ}{3L}\left[2\varphi_B + 3\frac{V}{3L}\right]$$

$$\Phi_C^{(2)} = \frac{2EJ}{3L}\left[\varphi_B + 3\frac{V}{3L}\right]$$

$$\frac{EJ}{L}\begin{bmatrix} 3{,}333 & -0{,}833 \\ -0{,}833 & 3{,}944 \end{bmatrix}\begin{bmatrix} \varphi_B \\ \frac{V}{L} \end{bmatrix} - \begin{bmatrix} -1 \\ \frac{3}{2} \end{bmatrix}\frac{EJ}{L} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\varphi_B = -0{,}216 \quad ; \quad \frac{V}{L} = 0{,}335$$

$$w_{BC}(y) = C_0 + C_1 y + C_2 y^2 + C_3 y^3$$

$w_{BC}(0) = 0{,}335L$

$w'_{BC}(0) = -0{,}216$

$w_{BC}(3L) = 0$

$w'_{BC}(3L) = 0$

$\longrightarrow$

$$w_{BC}(y) = L\cdot\left(0{,}335 - 0{,}216\frac{y}{L} + 0{,}032\frac{y^2}{L^2} + 0{,}001\frac{y^3}{L^3}\right)$$

$$y_* = 0$$

opracował

G. Dzierżanowski