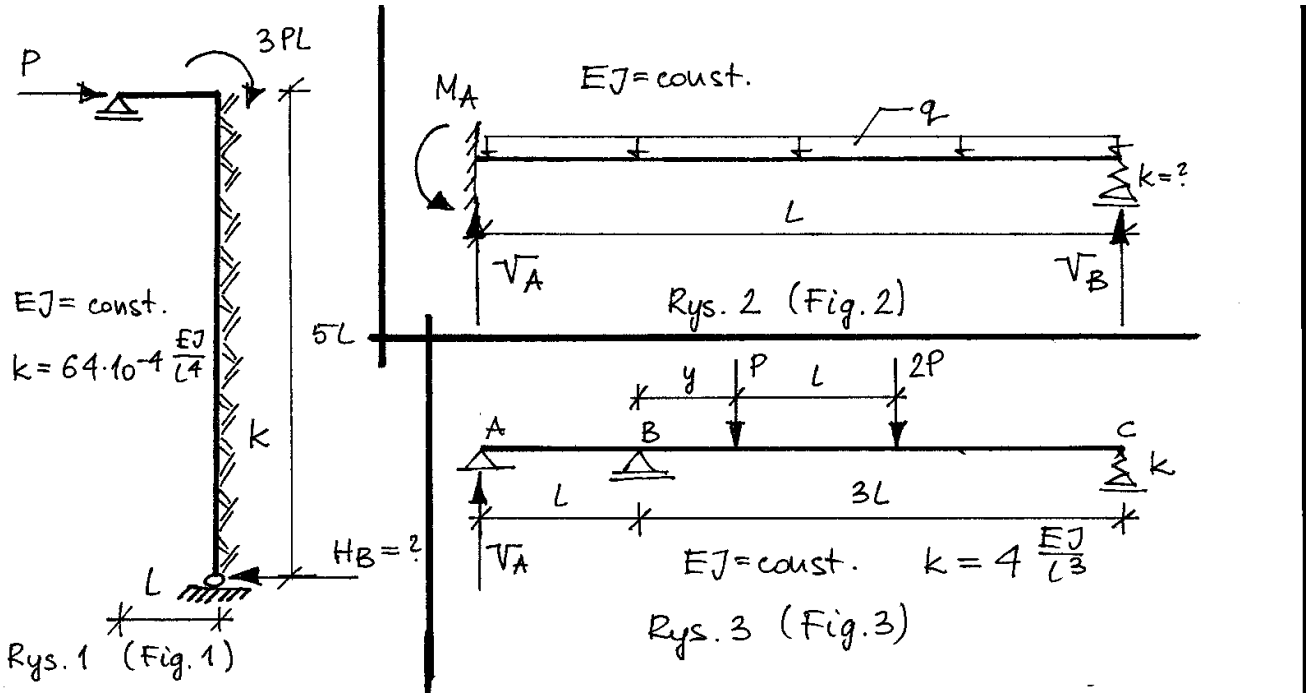
Egzamin z Mechaniki Konstrukcji, 2.09.2015
Exam on the Mechanics of Structures

NAZWISKO, Imię LAST NAME, First Name				
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egzaminu	ocena łączna

Zadanie 1 (Rys. 1) **Problem #1 (Fig. 1)**
Oblicz reakcję H_B korzystając z tw. Bettiego.
Calculate reaction H_B by the Betti Theorem.

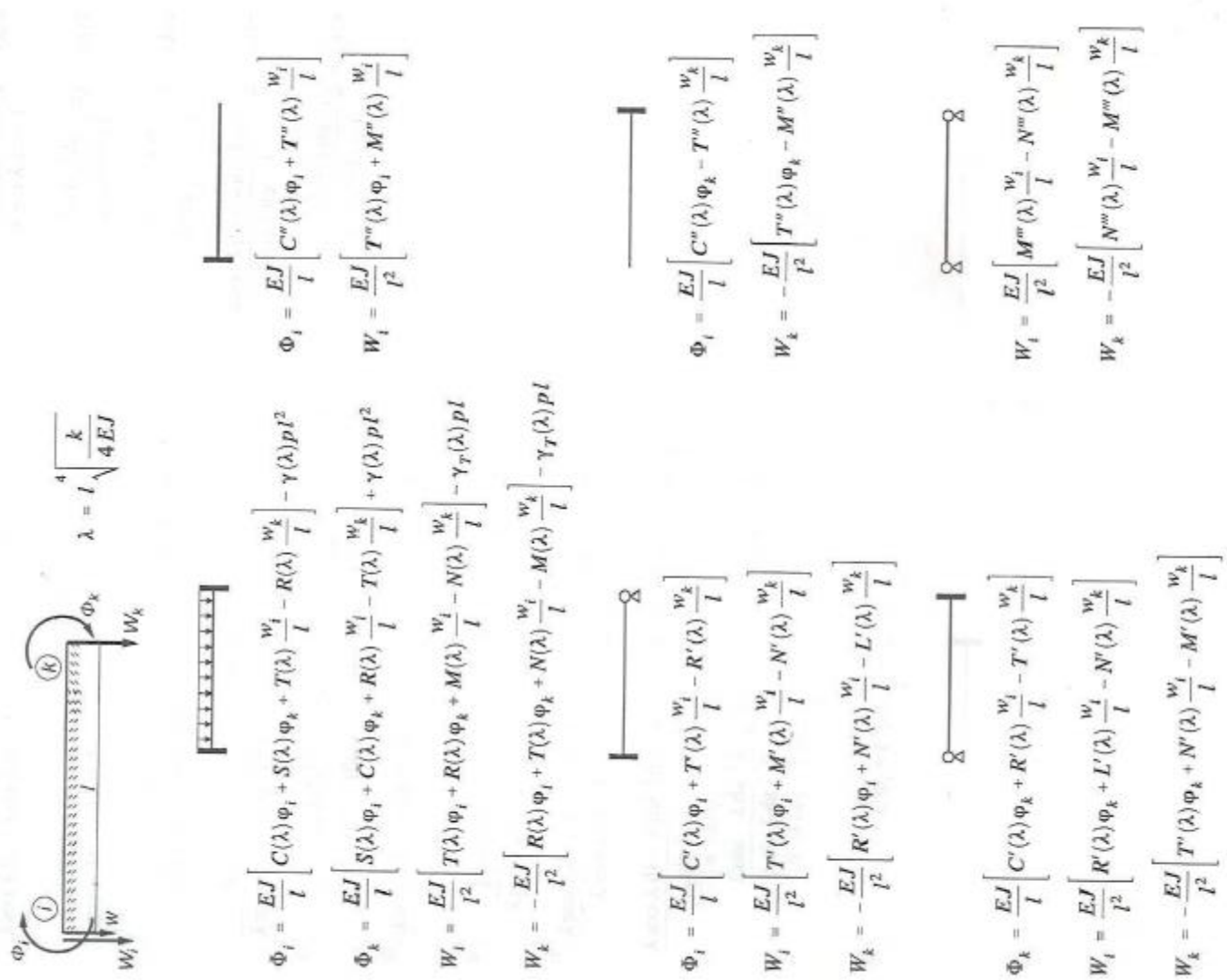
Zadanie 2 (Rys. 2) **Problem #2 (Fig. 2)**
Oblicz wartość k, dla której wartość momentu $M_A = 0.25\ ql^2$. Następnie oblicz wartości V_A oraz V_B.
Calculate the value of k for which the moment $M_A = 0.25\ ql^2$. Next, calculate V_A and V_B.

Zadanie 3 (Rys. 3) **Problem #3 (Fig. 3)**
Wyznacz położenie układu sił (P, $2P$) w przedziale B-C tak, aby wartość bezwzględna $|V_A|$ była największa.
Find the position of force system (P, $2P$) between points B and C for which the absolute value $|V_A|$ is maximal.

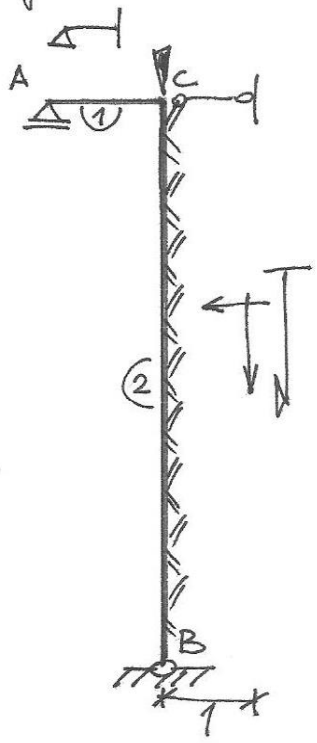


λ	C(λ)	S(λ)	T(λ)	R(λ)	M(λ)	N(λ)	C'(λ)	T'(λ)	R'(λ)
0,0	**4,000**	**2,000**	**6,000**	**6,000**	**12,000**	**12,000**	**3,000**	**3,000**	**3,000**
0,1	4,000	2,000	6,000	6,000	12,000	12,000	3,000	3,000	3,000
0,2	4,000	2,000	6,000	6,000	12,002	11,999	3,000	3,001	3,000
0,3	4,000	2,000	6,002	5,999	12,012	11,996	3,001	3,003	2,999
0,4	4,001	1,999	6,005	5,997	12,038	11,987	3,002	3,009	2,996
0,5	**4,002**	**1,998**	**6,013**	**5,992**	**12,093**	**11,968**	**3,005**	**3,021**	**2,990**
0,6	4,005	1,996	6,027	5,984	12,192	11,933	3,010	3,044	2,980
0,7	4,009	1,993	6,050	5,970	12,356	11,877	3,018	3,082	2,962
0,8	4,016	1,988	6,086	5,949	12,608	11,790	3,031	3,140	2,936
0,9	4,025	1,981	6,137	5,919	12,972	11,665	3,050	3,223	2,898
1,0	**4,038**	**1,972**	**6,208**	**5,877**	**13,480**	**11,491**	**3,075**	**3,338**	**2,846**
1,1	4,055	1,959	6,304	5,821	14,163	11,258	3,109	3,492	2,776
1,2	4,078	1,942	6,429	5,748	15,056	10,956	3,153	3,692	2,687
1,3	4,107	1,920	6,589	5,656	16,197	10,573	3,209	3,944	2,575
1,4	4,143	1,894	6,787	5,541	17,624	10,100	3,277	4,254	2,438
1,5	**4,186**	**1,862**	**7,030**	**5,402**	**19,377**	**9,526**	**3,359**	**4,629**	**2,275**
1,6	4,239	1,823	7,323	5,236	21,498	8,844	3,455	5,071	2,086
1,7	4,301	1,778	7,670	5,041	24,026	8,049	3,566	5,586	1,871
1,8	4,373	1,726	8,075	4,818	27,000	7,136	3,693	6,174	1,632
1,9	4,456	1,666	8,541	4,564	30,459	6,106	3,833	6,835	1,370
2,0	**4,550**	**1,600**	**9,073**	**4,280**	**34,438**	**4,962**	**3,988**	**7,568**	**1,091**

λ	M'(λ)	N'(λ)	L'(λ)	C''(λ)	T''(λ)	M''(λ)	M'''(λ)	N'''(λ)
0,0	**3,000**	**3,000**	**3,000**	**0,000**	**0,000**	**0,000**	**0,000**	**0,000**
0,1	3,000	3,000	3,000	0,000	0,000	0,000	0,000	0,000
0,2	3,003	2,999	3,002	0,002	0,003	0,006	0,002	-0,001
0,3	3,016	2,995	3,008	0,011	0,016	0,032	0,011	-0,005
0,4	3,050	2,986	3,024	0,034	0,051	0,102	0,034	-0,017
0,5	**3,121**	**2,965**	**3,059**	**0,082**	**0,123**	**0,247**	**0,083**	**-0,042**
0,6	3,251	2,928	3,122	0,166	0,250	0,505	0,172	-0,086
0,7	3,465	2,867	3,226	0,298	0,449	0,918	0,318	-0,158
0,8	3,793	2,774	3,385	0,484	0,734	1,520	0,541	-0,268
0,9	4,267	2,639	3,615	0,726	1,107	2,340	0,861	-0,424
1,0	**4,925**	**2,454**	**3,934**	**1,017**	**1,563**	**3,394**	**1,301**	**-0,635**
1,1	5,807	2,209	4,363	1,342	2,087	4,688	1,884	-0,910
1,2	6,954	1,893	4,920	1,686	2,658	6,225	2,630	-1,253
1,3	8,408	1,500	5,626	2,031	3,258	8,008	3,560	-1,665
1,4	10,213	1,021	6,503	2,363	3,871	10,052	4,689	-2,144
1,5	**12,408**	**0,455**	**7,571**	**2,675**	**4,492**	**12,380**	**6,029**	**-2,681**
1,6	15,031	-0,200	8,847	2,963	5,119	15,027	7,587	-3,263
1,7	18,117	-0,941	10,350	3,228	5,756	18,031	9,368	-3,872
1,8	21,694	-1,759	12,093	3,472	6,411	21,438	11,372	-4,487
1,9	25,786	-2,642	14,088	3,700	7,092	25,290	13,599	-5,085
2,0	**30,412**	**-3,573**	**16,347**	**3,915**	**7,807**	**29,631**	**16,049**	**-5,642**



Egzamin z MK IPB, 2.09.2015, zadanie 1

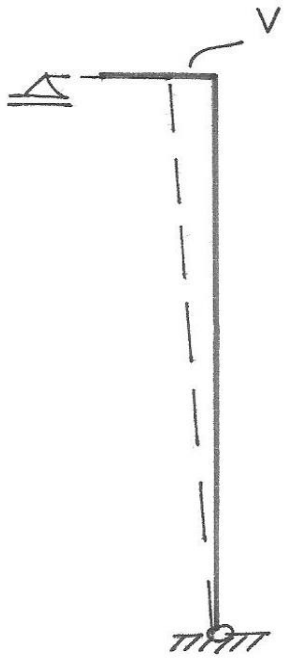


$$q = \begin{bmatrix} \varphi_C \\ \frac{v}{l} \end{bmatrix}$$

$$\lambda^4 = \frac{kl^4}{4EJ}$$

$$= 16 \cdot 10^{-4}$$

$$\lambda = 0{,}2$$



Kąt obrotu φ_C jest zgodny z ruchem wskazówek zegara
Rotation φ_C is clock-wise hence

$$-P \cdot v + 3Pl \cdot \varphi_C - H_B \cdot 1 = 0 \quad \text{(tw. Bettiego)}$$

$$H_B = -P \cdot v + 3Pl \cdot \varphi_C$$

$$\begin{cases} \Phi_C^{(1)} + \Phi_C^{(2)} = 0 \\ W_C^{(2)} \cdot \bar{V} = 0 \end{cases}$$

$$\Phi_C^{(1)} = \frac{3EJ}{l} \varphi_C$$

$$\Phi_C^{(2)} = \frac{EJ}{5l}\left[C'(5\lambda)\varphi_C + T'(5\lambda)\frac{v}{5l}\right] + \overset{\circ}{\Phi}{}_C^{(2)}$$

$$\overset{\circ}{\Phi}{}_C^{(2)} = \frac{EJ}{5l}\left[R'(5\lambda) \cdot \frac{1}{5l}\right]$$

$$W_C^{(2)} = \frac{EJ}{(5l)^2}\left[T'(5\lambda)\varphi_C + M'(5\lambda)\frac{v}{5l}\right] + \overset{\circ}{W}{}_C^{(2)}$$

$$\overset{\circ}{W}{}_C^{(2)} = \frac{EJ}{(5l)^2}\left[N'(5\lambda) \cdot \frac{1}{5l}\right]$$

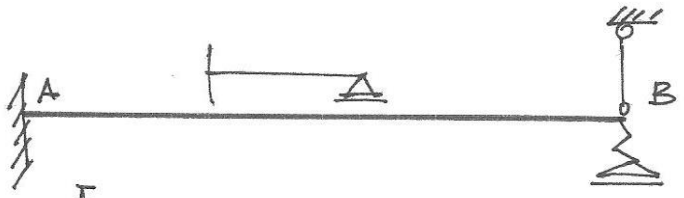
$$\begin{bmatrix} 3{,}615 & 0{,}133 \\ 0{,}133 & 0{,}039 \end{bmatrix} \begin{bmatrix} \varphi_C \\ \frac{v}{l} \end{bmatrix} + \begin{bmatrix} 0{,}114 \\ 0{,}02 \end{bmatrix} \frac{1}{l} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\varphi_C = -0{,}015 \cdot \frac{1}{l} \qquad \frac{v}{l} = -0{,}45 \cdot \frac{1}{l}$$

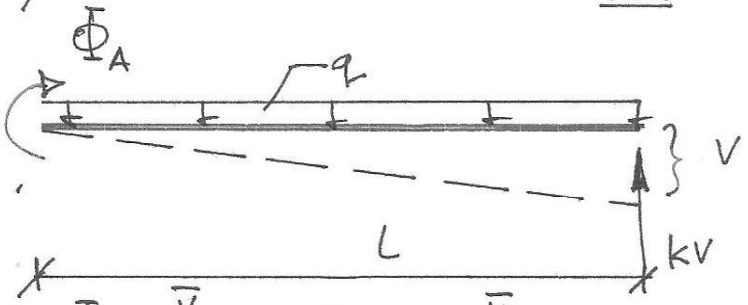
$$H_B = P \cdot 0{,}45 - 3 \cdot P \cdot 0{,}015 = 0{,}405\,P\,.$$

opracował: G. Dzierżanowski

Egzamin z MK IPB, 2.09.2015, zadanie 2



$$q = \left[\frac{V}{l}\right]$$



$$\Phi_A \cdot \frac{\bar{V}}{l} - kV\bar{V} + ql \cdot \frac{\bar{V}}{2} = 0$$

$$\Phi_A - kVl + \frac{1}{2}ql^2 = 0$$

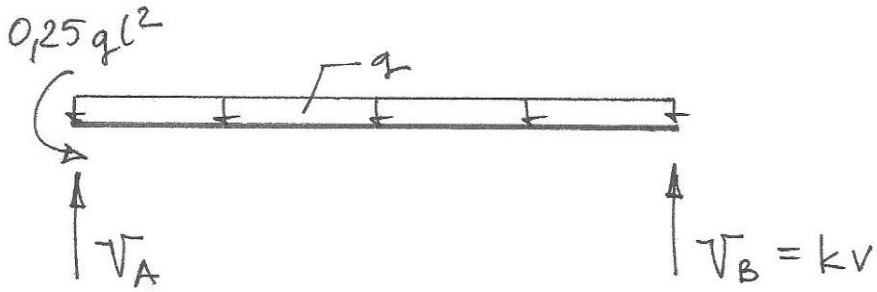
$$\Phi_A = \frac{3EJ}{l}\left[-\frac{V}{l}\right] - \frac{1}{8}ql^2$$

$$\frac{EJ}{l}(3+\tau)\frac{V}{l} - \frac{3}{8}ql^2 = 0 \quad , \quad \tau = \frac{kl^3}{EJ}$$

$$\frac{V}{l} = \frac{3}{8}\frac{ql^3}{EJ} \cdot \frac{1}{3+\tau}$$

$$M_A = -\Phi_A = \frac{12+\tau}{24+8\tau}ql^2$$

$$M_A = 0{,}25ql^2 \iff \tau = 6 \quad , \quad k = \frac{6EJ}{l^3}$$



$$V_B = k \cdot V = \frac{1}{4}ql$$

$$V_A = ql - V_B = \frac{3}{4}ql$$

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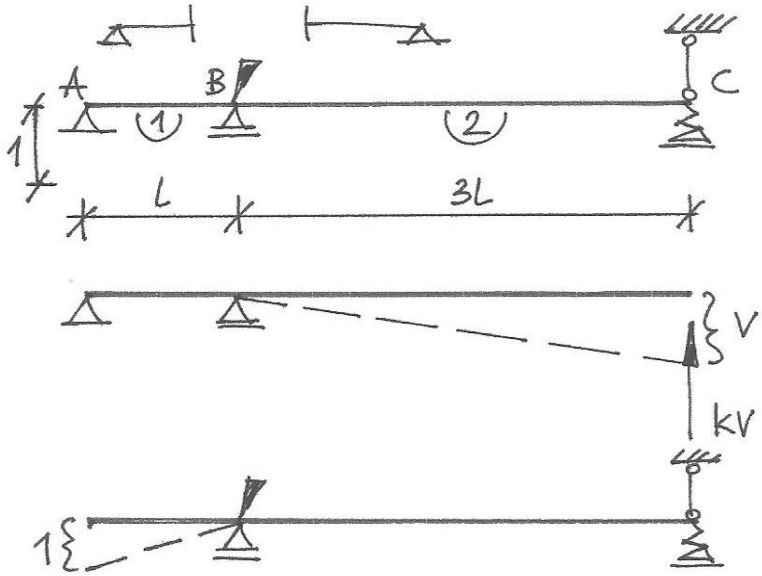
Egzamin z MK IPB, 2.09.2015, zadanie 3

W rozwiązaniu korzystamy z tw. Bettiego.

The solution is based on the Betti theorem.



$$q = \begin{bmatrix} \varphi_B \\ \frac{V}{L} \end{bmatrix}$$

$$\tau = 4$$

$$\overset{\circ}{\Phi}{}^{(1)}_{B} = \frac{3EJ}{L}\begin{bmatrix} 1 \\ \frac{1}{L} \end{bmatrix}$$

$$\begin{cases} \Phi^{(1)}_B + \Phi^{(2)}_B = 0 \\ \Phi^{(2)}_B \cdot \frac{\overline{V}}{3L} - k\overline{V}\overline{V} = 0 \end{cases}$$

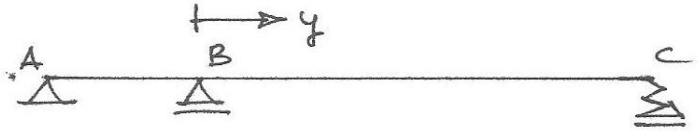
$$\Phi^{(1)}_B = \frac{3EJ}{L}\varphi_B + \overset{\circ}{\Phi}{}^{(1)}_{B}$$

$$\Phi^{(2)}_B = \frac{3EJ}{3L}\left[\varphi_B - \frac{V}{3L}\right]$$

$$\frac{EJ}{L}\begin{bmatrix} 4 & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{9}+4 \end{bmatrix}\begin{bmatrix} \varphi_B \\ \frac{V}{L} \end{bmatrix} + \frac{EJ}{L^2}\begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\varphi_B = -0{,}176 \cdot \frac{1}{L} \qquad \frac{V}{L} = -0{,}106 \cdot \frac{1}{L}$$

$$w_{BC}(y) = C_0 + C_1 y + C_2 y^2 + C_3 y^3 \quad , \quad y \in [0, 3L]$$



$$w_{BC}(0) = 0$$
$$w'_{BC}(0) = \varphi_B$$
$$w_{BC}(3L) = V$$
$$w''_{BC}(3L) = 0$$

$$\rightarrow$$

$$w_{BC}(y) = L \cdot \left(-0{,}176 \cdot \frac{y}{L} + 0{,}37\frac{y^2}{L^2} - 0{,}04\frac{y^3}{L^3}\right)$$

$$T_A(y) = P \cdot w_{BC}(y) + 2P \cdot w_{BC}(y+L)$$

$$y_* = 1{,}316\,L\,.$$

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