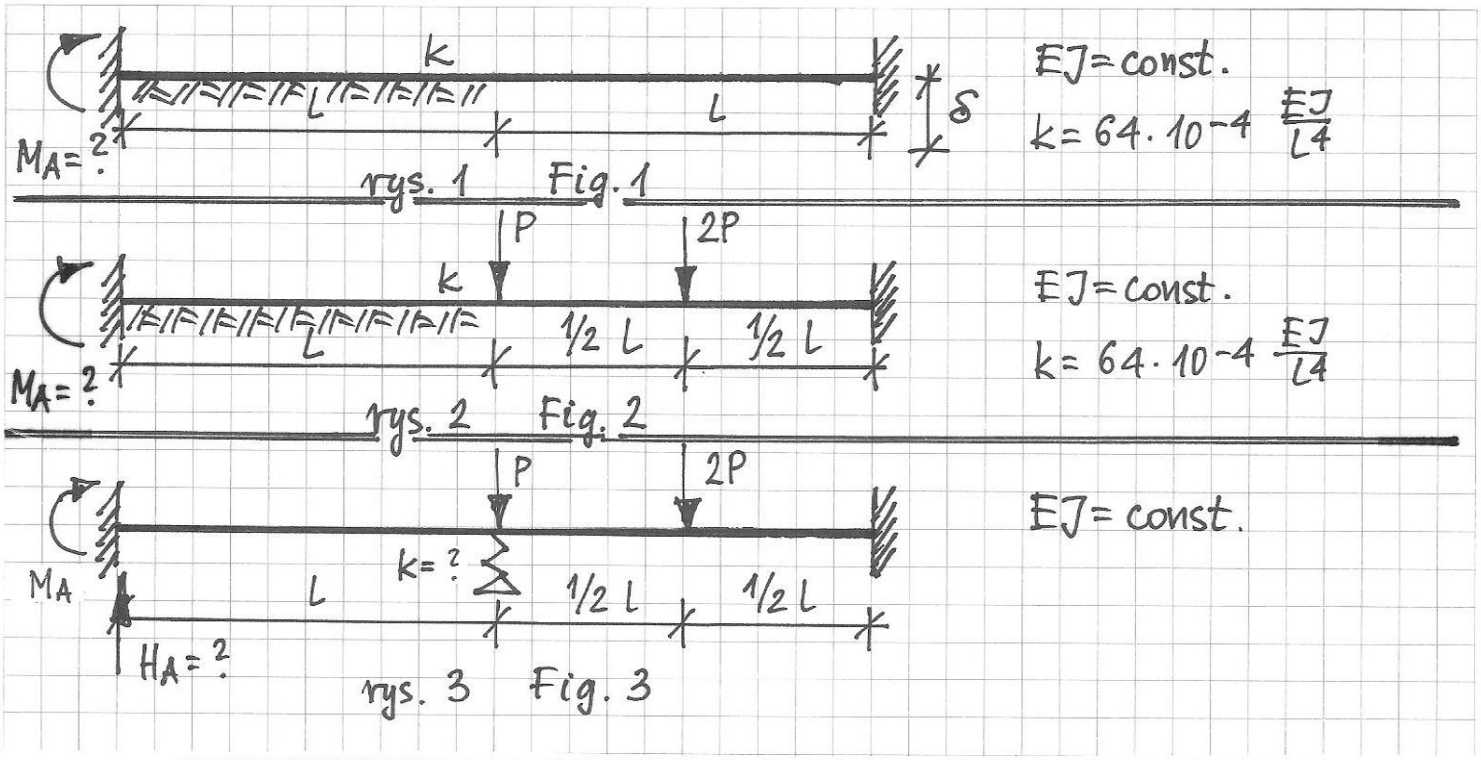
Egzamin z Mechaniki Konstrukcji, 27.06.2015
Exam on the Mechanics of Structures

NAZWISKO, Imię LAST NAME, First Name				
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egzaminu	ocena łączna

Zadanie 1 (Rys. 1) **Problem #1 (Fig. 1)**
Oblicz wartość momentu M_A korzystając z tw. Bettiego.
Calculate the moment M_A by the Betti Theorem.

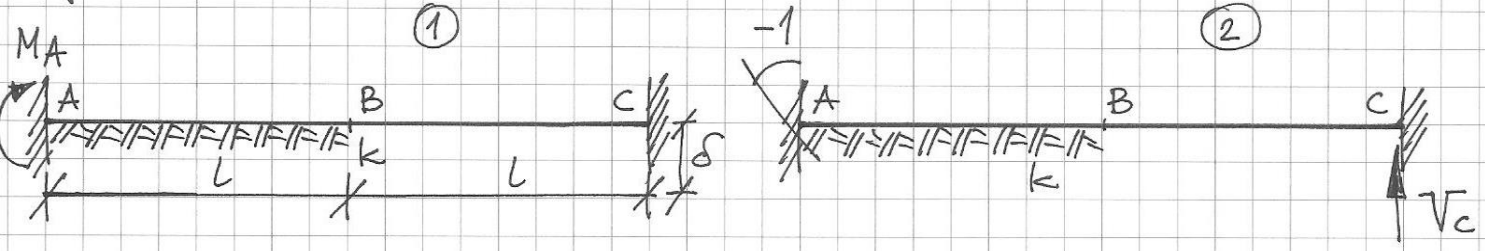
Zadanie 2 (Rys. 2) **Problem #2 (Fig. 2)**
Oblicz wartość momentu M_A korzystając z tw. Bettiego.
Calculate the moment M_A by the Betti Theorem.

Zadanie 3 (Rys. 3) **Problem #3 (Fig. 3)**
Oblicz wartość k taką, że moment $M_A = 0$. Następnie oblicz H_A.
Calculate the value of k such that the moment $M_A = 0$. Next, calculate H_A.



λ	C(λ)	S(λ)	T(λ)	R(λ)	M(λ)	N(λ)	C'(λ)	T'(λ)	R'(λ)
0,0	**4,000**	**2,000**	**6,000**	**6,000**	**12,000**	**12,000**	**3,000**	**3,000**	**3,000**
0,1	4,000	2,000	6,000	6,000	12,000	12,000	3,000	3,000	3,000
0,2	4,000	2,000	6,000	6,000	12,002	11,999	3,000	3,001	3,000
0,3	4,000	2,000	6,002	5,999	12,012	11,996	3,001	3,003	2,999
0,4	4,001	1,999	6,005	5,997	12,038	11,987	3,002	3,009	2,996
0,5	**4,002**	**1,998**	**6,013**	**5,992**	**12,093**	**11,968**	**3,005**	**3,021**	**2,990**
0,6	4,005	1,996	6,027	5,984	12,192	11,933	3,010	3,044	2,980
0,7	4,009	1,993	6,050	5,970	12,356	11,877	3,018	3,082	2,962
0,8	4,016	1,988	6,086	5,949	12,608	11,790	3,031	3,140	2,936
0,9	4,025	1,981	6,137	5,919	12,972	11,665	3,050	3,223	2,898
1,0	**4,038**	**1,972**	**6,208**	**5,877**	**13,480**	**11,491**	**3,075**	**3,338**	**2,846**
1,1	4,055	1,959	6,304	5,821	14,163	11,258	3,109	3,492	2,776
1,2	4,078	1,942	6,429	5,748	15,056	10,956	3,153	3,692	2,687
1,3	4,107	1,920	6,589	5,656	16,197	10,573	3,209	3,944	2,575
1,4	4,143	1,894	6,787	5,541	17,624	10,100	3,277	4,254	2,438
1,5	**4,186**	**1,862**	**7,030**	**5,402**	**19,377**	**9,526**	**3,359**	**4,629**	**2,275**
1,6	4,239	1,823	7,323	5,236	21,498	8,844	3,455	5,071	2,086
1,7	4,301	1,778	7,670	5,041	24,026	8,049	3,566	5,586	1,871
1,8	4,373	1,726	8,075	4,818	27,000	7,136	3,693	6,174	1,632
1,9	4,456	1,666	8,541	4,564	30,459	6,106	3,833	6,835	1,370
2,0	**4,550**	**1,600**	**9,073**	**4,280**	**34,438**	**4,962**	**3,988**	**7,568**	**1,091**

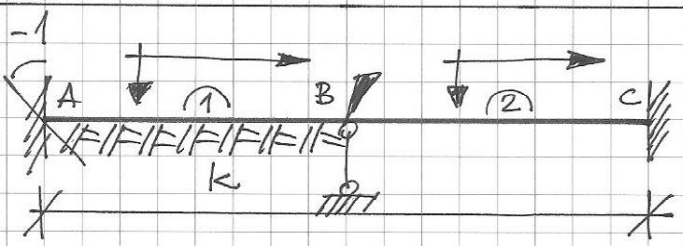
Egzamin z MK3 (IPB) 27.06.2015 Zadanie 1



Na podstawie tw. Bettiego:
By the Betti Theorem:

$$M_A \cdot (-1) = -V_C \cdot \delta$$
$$M_A = \delta \cdot V_C$$

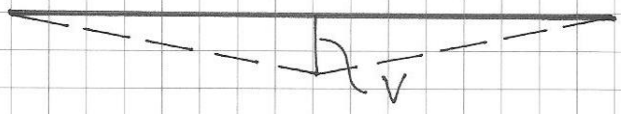
gdzie V_C jest reakcją pionową w punkcie C stowarzyszoną z kątem obrotu = -1 w punkcie A.
where V_C denotes the vertical reaction at C related to rotation angle = -1 at point A.



$EJ = const.$ $\quad k = 64 \cdot 10^{-4} \frac{EJ}{l^4}$

$\lambda^4 = \frac{kl^4}{4EJ}$ $\quad \lambda = 0,2$

$$q = \left[\varphi_B \quad \frac{V}{l} \right]^T$$



$$\begin{cases} \Phi_B^{(1)} + \Phi_B^{(2)} = 0 \\ W_B^{(1)} \cdot \bar{V} + W_B^{(2)} \cdot \bar{V} = 0 \end{cases}$$

$$\Phi_B^{(1)} = \frac{EJ}{l}\left[C(0,2)\varphi_B - T(0,2)\frac{V}{l} \right] + \overset{\circ}{\Phi}{}_B^{(1)}$$

$$\Phi_B^{(2)} = \frac{2EJ}{l}\left[2\varphi_B + 3\frac{V}{l} \right]$$

$$W_B^{(1)} = -\frac{EJ}{l^2}\left[T(0,2)\varphi_B - M(0,2)\frac{V}{l} \right] + \overset{\circ}{W}{}_B^{(1)}$$

$$W_B^{(2)} = \frac{6EJ}{l^2}\left[\varphi_B + 2\frac{V}{l} \right]$$

$$\overset{\circ}{\Phi}{}_B^{(1)} = \frac{EJ}{l}\left[S(0,2) \cdot (-1) \right]$$

$$\overset{\circ}{W}{}_B^{(1)} = -\frac{EJ}{l^2}\left[R(0,2) \cdot (-1) \right]$$

$$\begin{cases} 8\varphi_B - 2 = 0 \\ 24,002\frac{V}{l} + 6 = 0 \end{cases} \rightarrow \begin{array}{l} \varphi_B = \frac{1}{4} = 0,25 \\ \frac{V}{l} = -0,25 \end{array}$$

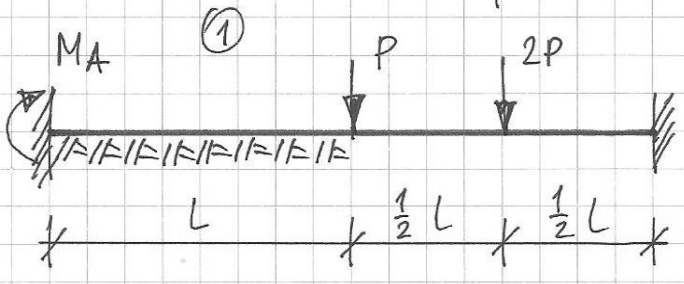
$$V_C = -W_C^{(2)} = \frac{6EJ}{l^2}\left[\varphi_B + 2\frac{V}{l} \right] = 1,5\frac{EJ}{l^2}$$

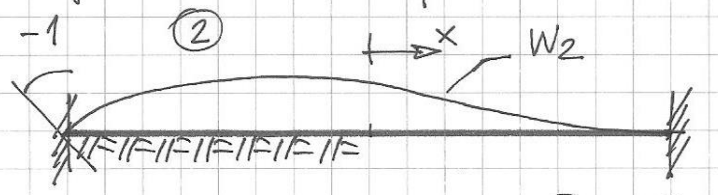
$$M_A = 1,5\frac{EJ}{l^2} \cdot \delta$$

opracował:
G. Dzierżanowski

Egzamin z MK3 (IPB) 27.06.2015 Zadanie 2

W rozwiązaniu zadania 2 można skorzystać z zadania 1
Solution of the problem #1 may be used in problem #2



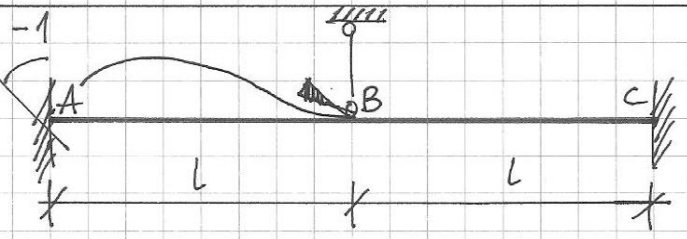


$W_2 = W_2(x)\,, \quad x \in [0, L]$

Z tw. Bettiego :
By the Betti Theorem:

$$M_A \cdot (-1) + P \cdot W_2(0) + 2P \cdot W_2\left(\tfrac{1}{2}L\right) = 0$$

$$M_A = P \cdot W_2(0) + 2P \cdot W_2\left(\tfrac{1}{2}L\right)$$



$$q = \left[\varphi_B \quad \frac{V}{L}\right]^T$$

Rozwiązanie zadania 1 :
Solution to problem #1:

$\varphi_B = 0{,}25 \qquad \frac{V}{L} = -0{,}25$

$$W_2(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3$$

$W_2(0) = V$
$W_2'(0) = \varphi_B$
$W_2(L) = 0$
$W_2'(L) = 0$

$$\rightarrow \quad W_2(x) = -0{,}25\,L\left(1 - \frac{x}{L} - \frac{x^2}{L^2} + \frac{x^3}{L^3}\right)$$

$$M_A = -0{,}25\,PL - 2P \cdot 0{,}093\,L = -0{,}436\,PL$$

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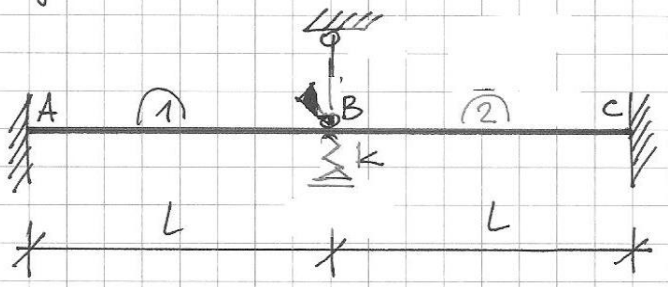
Egzamin z MK3 (IPB) 27.06.2015 Zadanie 3



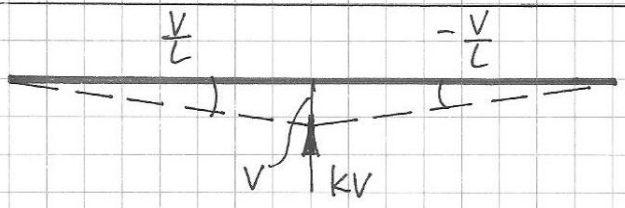
$EJ = const.$

$$\mathbf{q} = \left[\varphi_B \quad \frac{V}{L}\right]^T$$

$$\tau = \frac{kL^3}{EJ}$$

$$\overset{\circ}{\Phi}{}^{(2)}_B = -\frac{PL}{4} \qquad \overset{\circ}{\Phi}{}^{(2)}_C = \frac{PL}{4}$$

Momenty wyjściowe Tatwo obliczyć korzystając z Metody Sił.
Initial moments can be easily calculated by the Force Method.



1) $\Phi^{(1)}_B + \Phi^{(2)}_B = 0$

2) $\left[\Phi^{(1)}_A + \Phi^{(1)}_B\right] \cdot \frac{V}{L} + \left[\Phi^{(2)}_B + \Phi^{(2)}_C\right] \cdot \left(-\frac{V}{L}\right)$

$$-kV\bar{V} + P\bar{V} + 2P \cdot \frac{1}{2}\bar{V} = 0$$

$$\Phi^{(1)}_A = \frac{2EJ}{L}\left[\varphi_B - 3\frac{V}{L}\right] \qquad \Phi^{(2)}_B = \frac{2EJ}{L}\left[2\varphi_B + 3\frac{V}{L}\right] + \overset{\circ}{\Phi}{}^{(2)}_B$$

$$\Phi^{(1)}_B = \frac{2EJ}{L}\left[2\varphi_B - 3\frac{V}{L}\right] \qquad \Phi^{(2)}_C = \frac{2EJ}{L}\left[\varphi_B + 3\frac{V}{L}\right] + \overset{\circ}{\Phi}{}^{(2)}_C$$

$$\begin{cases} 8\varphi_B - \dfrac{PL^2}{4EJ} = 0 \\ (24+\tau)\dfrac{V}{L} - 2\dfrac{PL^2}{EJ} = 0 \end{cases} \longrightarrow \quad \varphi_B = \frac{PL^2}{32EJ}, \quad \frac{V}{L} = \frac{2}{24+\tau}\frac{PL^2}{EJ}$$

$$M_A = \Phi^{(1)}_A = 0 \longrightarrow \frac{1}{32} - \frac{6}{24+\tau} = 0 \longrightarrow \tau = 168$$

$$H_A = -W^{(1)}_A = -\frac{6EJ}{L^2}\left[\varphi_B + 2\frac{V}{L}\right] = -\frac{5}{16}P$$

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