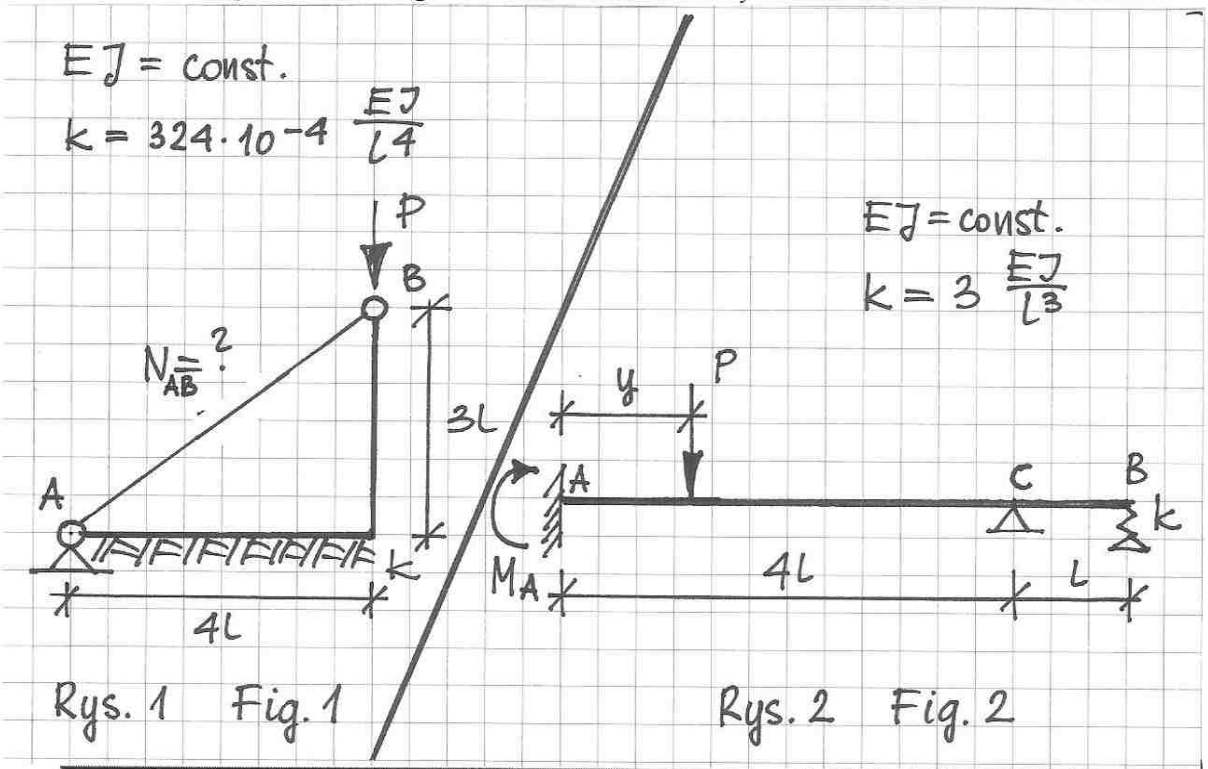
**Egzamin z Mechaniki Konstrukcji, 20.06.2015**
**Exam on the Mechanics of Structures**

| NAZWISKO, Imię<br>LAST NAME, First Name | | | | |
|---|---|---|---|---|
| ocena zadania 1 | ocena zadania 2 | ocena zadania 3 | ocena egzaminu | ocena łączna |
| | | | | |

**Zadanie 1 (Rys. 1)** **Problem #1 (Fig. 1)**
Oblicz siłę $N_{AB}$ w pręcie kratowym.
Calculate the force $N_{AB}$ in the truss member.

**Zadanie 2 (Rys. 2)** **Problem #2 (Fig. 2)**
Wyznacz położenie siły $P$ w przedziale $A$-$C$ tak, aby wartość bezwzględna $|M_A|$ była największa.
Find the position of force $P$ between points $A$ and $C$ for which the absolute value $|M_A|$ is maximal.

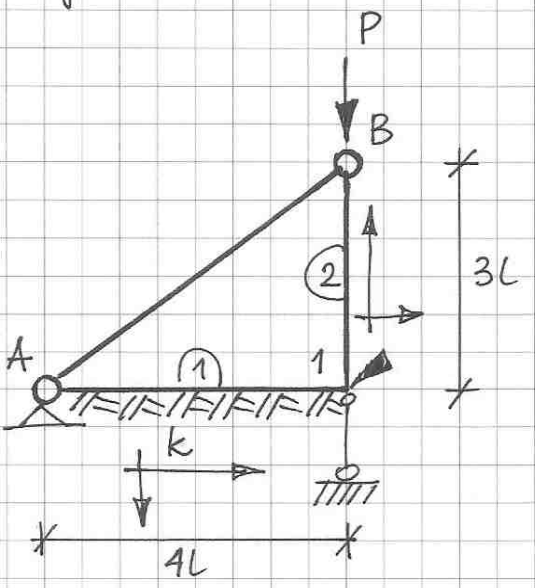
**Zadanie 3 (Rys. 2)** **Problem #3 (Fig. 2)**
Oblicz kąt obrotu w punkcie $B$ zakładając, że obciążenie $P$ działa w punkcie $y=2l$. Skorzystaj z tw. Maxwella-Mohra.
Calculate the rotation angle at $B$ assuming that $P$ loads the structure at $y=2l$. Use the Maxwell-Mohr formula.

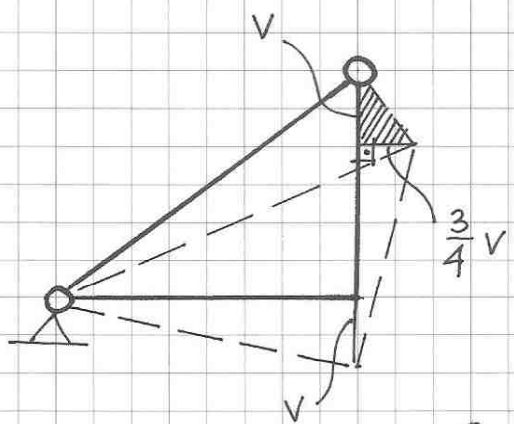



Rys. 1 Fig. 1 Rys. 2 Fig. 2

| λ | C(λ) | S(λ) | T(λ) | R(λ) | M(λ) | N(λ) | C'(λ) | T'(λ) | R'(λ) |
|---|---|---|---|---|---|---|---|---|---|
| **0,0** | **4,000** | **2,000** | **6,000** | **6,000** | **12,000** | **12,000** | **3,000** | **3,000** | **3,000** |
| 0,1 | 4,000 | 2,000 | 6,000 | 6,000 | 12,000 | 12,000 | 3,000 | 3,000 | 3,000 |
| 0,2 | 4,000 | 2,000 | 6,000 | 6,000 | 12,002 | 11,999 | 3,000 | 3,001 | 3,000 |
| 0,3 | 4,000 | 2,000 | 6,002 | 5,999 | 12,012 | 11,996 | 3,001 | 3,003 | 2,999 |
| 0,4 | 4,001 | 1,999 | 6,005 | 5,997 | 12,038 | 11,987 | 3,002 | 3,009 | 2,996 |
| **0,5** | **4,002** | **1,998** | **6,013** | **5,992** | **12,093** | **11,968** | **3,005** | **3,021** | **2,990** |
| 0,6 | 4,005 | 1,996 | 6,027 | 5,984 | 12,192 | 11,933 | 3,010 | 3,044 | 2,980 |
| 0,7 | 4,009 | 1,993 | 6,050 | 5,970 | 12,356 | 11,877 | 3,018 | 3,082 | 2,962 |
| 0,8 | 4,016 | 1,988 | 6,086 | 5,949 | 12,608 | 11,790 | 3,031 | 3,140 | 2,936 |
| 0,9 | 4,025 | 1,981 | 6,137 | 5,919 | 12,972 | 11,665 | 3,050 | 3,223 | 2,898 |
| **1,0** | **4,038** | **1,972** | **6,208** | **5,877** | **13,480** | **11,491** | **3,075** | **3,338** | **2,846** |
| 1,1 | 4,055 | 1,959 | 6,304 | 5,821 | 14,163 | 11,258 | 3,109 | 3,492 | 2,776 |
| 1,2 | 4,078 | 1,942 | 6,429 | 5,748 | 15,056 | 10,956 | 3,153 | 3,692 | 2,687 |
| 1,3 | 4,107 | 1,920 | 6,589 | 5,655 | 16,197 | 10,573 | 3,209 | 3,944 | 2,575 |
| 1,4 | 4,143 | 1,894 | 6,787 | 5,541 | 17,624 | 10,100 | 3,277 | 4,254 | 2,438 |
| **1,5** | **4,186** | **1,862** | **7,030** | **5,402** | **19,377** | **9,526** | **3,359** | **4,629** | **2,275** |
| 1,6 | 4,239 | 1,823 | 7,323 | 5,236 | 21,498 | 8,844 | 3,455 | 5,071 | 2,086 |
| 1,7 | 4,301 | 1,778 | 7,670 | 5,041 | 24,026 | 8,049 | 3,566 | 5,586 | 1,871 |
| 1,8 | 4,373 | 1,726 | 8,075 | 4,818 | 27,000 | 7,136 | 3,693 | 6,174 | 1,632 |
| 1,9 | 4,456 | 1,666 | 8,541 | 4,564 | 30,459 | 6,105 | 3,833 | 6,835 | 1,370 |
| **2,0** | **4,550** | **1,600** | **9,073** | **4,280** | **34,438** | **4,962** | **3,988** | **7,568** | **1,091** |

Egzamin z MK3 (IPB) 20.06.2015 Zadanie 1




$EJ = \text{const.}$

$k = 324 \cdot 10^{-4} \frac{EJ}{L^4}$

$\lambda^4 = \frac{kL^4}{4EJ}$

$\lambda = 0{,}3$

$\lambda^{(1)} = 1{,}2$

$$q = \left[\varphi_1 \quad \frac{V}{L}\right]^T$$

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$$\begin{cases} \Phi_1^{(1)} + \Phi_1^{(2)} = 0 \\ W_1^{(1)} \cdot \overline{V} + W_B^{(2)} \cdot \frac{3}{4}\overline{V} = P \cdot \overline{V} \end{cases}$$

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$$\Phi_1^{(1)} = \frac{EJ}{4L}\left[C'(1,2)\varphi_1 - T'(1,2)\cdot\frac{V}{4L}\right]$$

$$\Phi_1^{(2)} = \frac{3EJ}{3L}\left[\varphi_1 - \frac{1}{3L}\cdot\frac{3}{4}V\right]$$

$$W_1^{(1)} = -\frac{EJ}{16L^2}\left[T'(1,2)\varphi_B - M'(1,2)\frac{V}{4L}\right]$$

$$W_B^{(2)} = -\frac{3EJ}{9L^2}\left[\varphi_1 - \frac{V}{4L}\right]$$

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$$\begin{cases} 1{,}788\,\varphi_1 - 0{,}481\,\frac{V}{L} = 0 \\ -0{,}481\,\varphi_1 + 0{,}171\,\frac{V}{L} = \frac{PL^2}{EJ} \end{cases} \rightarrow \quad \varphi_1 = 6{,}414\,\frac{PL^2}{EJ}, \quad \frac{V}{L} = 23{,}859\,\frac{PL^2}{EJ}$$

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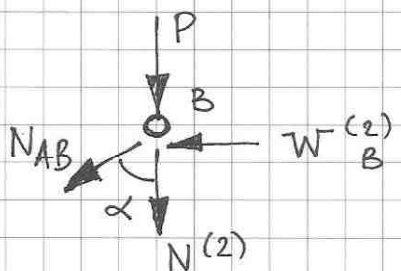
Wartość $N_{AB}$ oblicza się z warunku równowagi węzła B:
The value of $N_{AB}$ is calculated from the equilibrium condition at B:




$$N_{AB}\sin\alpha + W_B^{(2)} = 0$$

$$N_{AB} = -\frac{5}{4}W_B^{(2)} = 0{,}187P$$

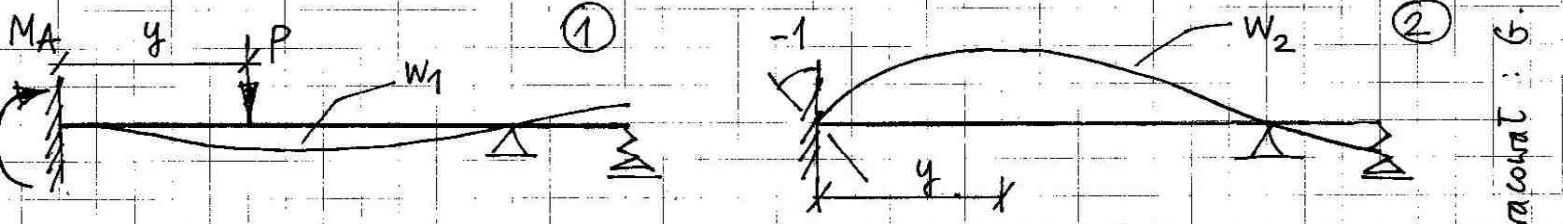
opracował: G. Dzierżanowski

Egzamin z MK3 (IPB) 20.06.2015 Zadanie 2

opracował: G. Dzierżanowski

Rozwiązanie z wykorzystaniem tw. Bettiego:

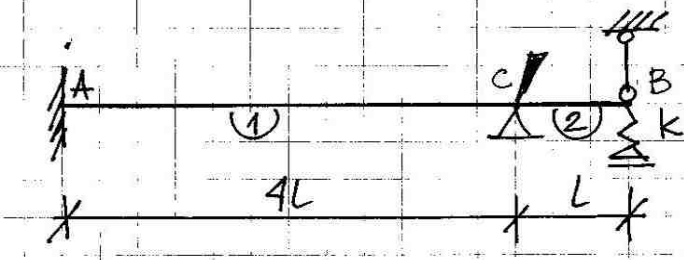
Solution by the Betti Theorem:




$$M_A(y)\cdot(-1) + P\cdot W_2(y) = 0 \rightarrow M_A(y) = P W_2(y)$$

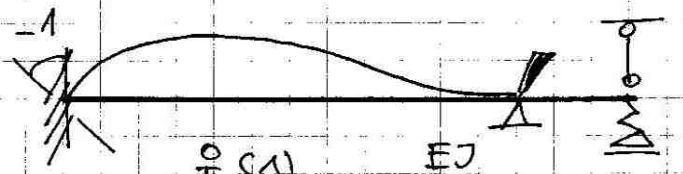
Celem zadania jest znalezienie postaci $W_2 = W_2(y)$ stowarzyszonej z obrotem podpory A o kąt równy $-1$.

The goal now is to find $W_2 = W_2(y)$ related to the rotation angle equal $-1$ at point A.




$EJ = \text{const.} \qquad k = 3\frac{EJ}{L^3}$

$$q = \left[\varphi_C \quad \frac{V}{L}\right]^T \qquad t = \frac{kL^3}{EJ}$$




$$\overset{\circ}{\Phi}{}_C^{(1)} = -\frac{EJ}{2L}$$


$$\begin{cases} \Phi_C^{(1)} + \Phi_C^{(2)} = 0 \\ \Phi_C^{(2)}\cdot\frac{V}{L} - kV\overline{V} = 0 \end{cases}$$

$$\Phi_C^{(1)} = \frac{2EJ}{4L}\left[2\varphi_C\right] + \overset{\circ}{\Phi}{}_C^{(1)}$$

$$\Phi_C^{(2)} = \frac{3EJ}{L}\left[\varphi_C - \frac{V}{L}\right]$$

$$\begin{cases} 4\varphi_C - 3\frac{V}{L} = \frac{1}{2} \\ -3\varphi_C + 6\frac{V}{L} = 0 \end{cases}$$

$$\varphi_C = \frac{1}{5} \qquad \frac{V}{L} = \frac{1}{10}$$

$$W_2(y) = C_0 + C_1 y + C_2 y^2 + C_3 y^3 \qquad y \in [0, 4L]$$

$$\begin{aligned} W(0) &= 0 \\ W'(0) &= -1 \\ W(4L) &= 0 \\ W'(4L) &= \frac{1}{5} \end{aligned} \rightarrow \begin{aligned} C_0 &= 0 & C_1 &= -1 \\ C_2 &= \frac{9}{20L} & C_3 &= -\frac{1}{20L^2} \end{aligned}$$

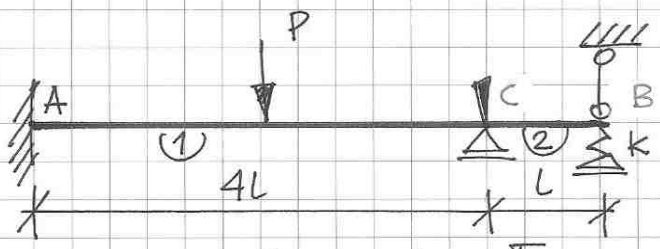
$$M_A(y) = -PL\left(\frac{y}{L} - \frac{9}{20}\frac{y^2}{L^2} + \frac{1}{20}\frac{y^3}{L^3}\right)$$

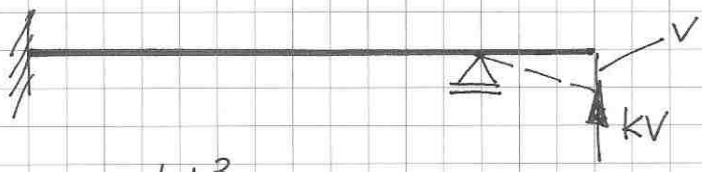
$$\frac{dM_A}{dy}(y) = -P\left(1 - \frac{9}{10}\frac{y}{L} + \frac{3}{20}\frac{y^2}{L^2}\right)$$

$$\frac{dM_A}{dy}(y) = 0 \rightarrow y^* = 1{,}472L \rightarrow M_A(y^*) = -0{,}656PL$$

Egzamin z MK3 (IPB) 20.06.2015 Zadanie 3

$EJ = \text{const.}$ $k = 3\frac{EJ}{L^3}$




$q = [\varphi_C \quad \frac{V}{L}]^T$ $\tau = \frac{kL^3}{EJ}$

opracował: G. Dzierżanowski

$$\begin{cases} \Phi_C^{(1)} + \Phi_C^{(2)} = 0 \\ \Phi_C^{(2)} \cdot \frac{\bar{V}}{L} - k V \bar{V} = 0 \end{cases}$$

$$\Phi_C^{(1)} = \frac{2EJ}{4L}[2\varphi_C] + \overset{\circ}{\Phi}{}_C^{(1)} \qquad \overset{\circ}{\Phi}{}_C^{(1)} = \frac{PL}{2}$$

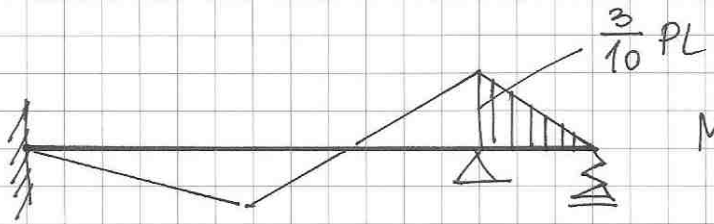
$$\Phi_C^{(2)} = \frac{3EJ}{L}\left[\varphi_C - \frac{V}{L}\right]$$

Moment wyjściowy $\overset{\circ}{\Phi}{}_C^{(1)}$ łatwo wyznaczyć Metodą Sił

Initial moment $\overset{\circ}{\Phi}{}_C^{(1)}$ can be easily found by The Force Method

$$\begin{cases} 4\varphi_C - 3\frac{V}{L} = -\frac{PL^2}{2EJ} \\ -3\varphi_C + 6\frac{V}{L} = 0 \end{cases} \rightarrow \varphi_C = -\frac{1}{5}\frac{PL^2}{EJ} \rightarrow \Phi_C^{(2)} = \frac{3}{10}PL$$

$$\frac{V}{L} = -\frac{1}{10}\frac{PL^2}{EJ}$$

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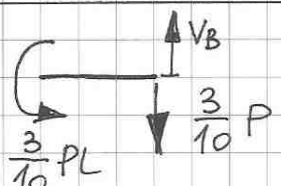
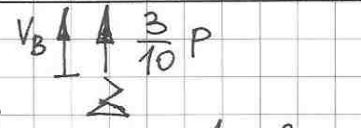
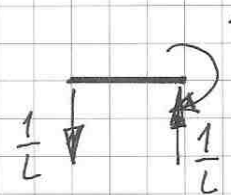



$$V_B = \frac{1}{k} \cdot \frac{3}{10}P$$

W obliczeniu $\varphi_B$ redukuje się konstrukcję do statycznie wyznaczalnej

In calculations of $\varphi_B$ the structure is reduced to statically determinate.


$$1 \cdot \varphi_B + \frac{1}{L} \cdot V_B =$$

$$= \frac{1}{EJ}\left[\frac{1}{2} \cdot L \cdot 1 \cdot \frac{1}{3} \cdot \frac{3}{10}PL\right]$$

$$\varphi_B = -\frac{1}{20}\frac{PL^2}{EJ}$$